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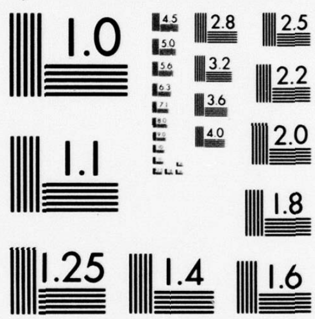

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"ANALYZE" — ANALYSIS OF AEROSPACE STRUCTURES WITH MEMBRANE ELEMENTS

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DECEMBER 1978

TECHNICAL REPORT AFFDL-TR-78-170
Final Report for May – August 1978

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AIR FORCE FLIGHT DYNAMICS LABORATORY
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This report has been reviewed by the Information Office (IO) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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This report contains documentation for the program 'ANALYZE'. The program library consists of a bar, a membrane triangle, a quadrilateral, and a shear panel. The equations of finite element analysis, element formulations, program organization, and subroutine descriptions provide a comprehensive theoretical background for the program. The input and output instructions together with the sample problem and the results should provide adequate information for the use of this program. (See reverse side)

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"ANALYZE" is an in-house program and can be used on INTERCOM for problems up to 150 to 200 degrees of freedom and a comparable number of elements. This program is extremely useful in training engineers in the use of finite element programs, in the development of finite element models of large aerospace structures, and in research in structural analysis and optimization.

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FOREWORD

This report is prepared as part of an in-house effort under Project 2401, Task No. 240102, "Design and Analysis Methods for Aerospace Vehicle Structures," and Work Unit 24010208, "Automated Design of Advanced Aerospace Structures." The work was carried out in the Design and Analysis Methods Group of the Analysis & Optimization Branch (FBR), Structural Mechanics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio.

The time period of the effort for this Technical Report is May 1978 - August 1978. The manuscript was originally released as AFFDL-FBR-TM-78-89 in August 1978.

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1. INTRODUCTION

The program "ANALYZE" was orginally developed for in-house studies in structural analysis and optimization. It is the basis for a number of programs such as "OPTSTAT"[1], OPTCOMP[2] and "DANALYZ"[3]. "ANALYZE" has been used by the authors in their consultation work on a number of Air Force projects. It has also been used as a demonstration program in structural analysis courses at the University of Dayton and the Air Force Institute of Technology. This program was distributed earlier with makeshift input and output instructions. These instructions did not include details of the theory nor the internal organization of the program. The purpose of this report is to generate comprehensive documentation for the "ANALYZE" program.

The program is based on the displacement method of finite element analysis[4-6]. In such an analysis the continuum is replaced by a discrete model consisting of a finite number of nodes connected by elements (See Figure 1). This discretization reduces the original differential equations of the continuum to a set of algebraic equations which can be solved much more readily on digital computers.

The program has basically four finite elements:

1. Bar (Axial Force Member)
2. Membrane Triangle
3. Membrane Quadrilateral
4. Shear Panel

The four elements and their local coordinate systems are shown in Figure 2. The bar is a constant strain line element and is equivalent to a rod

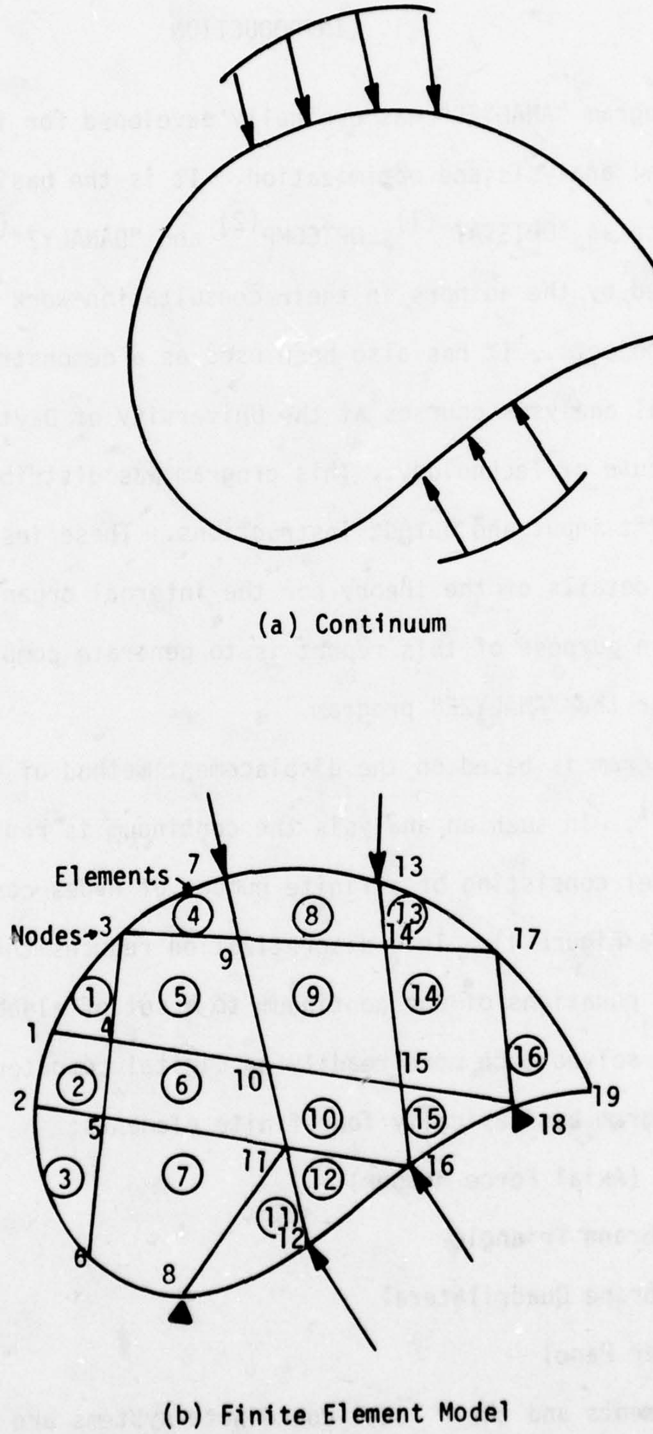
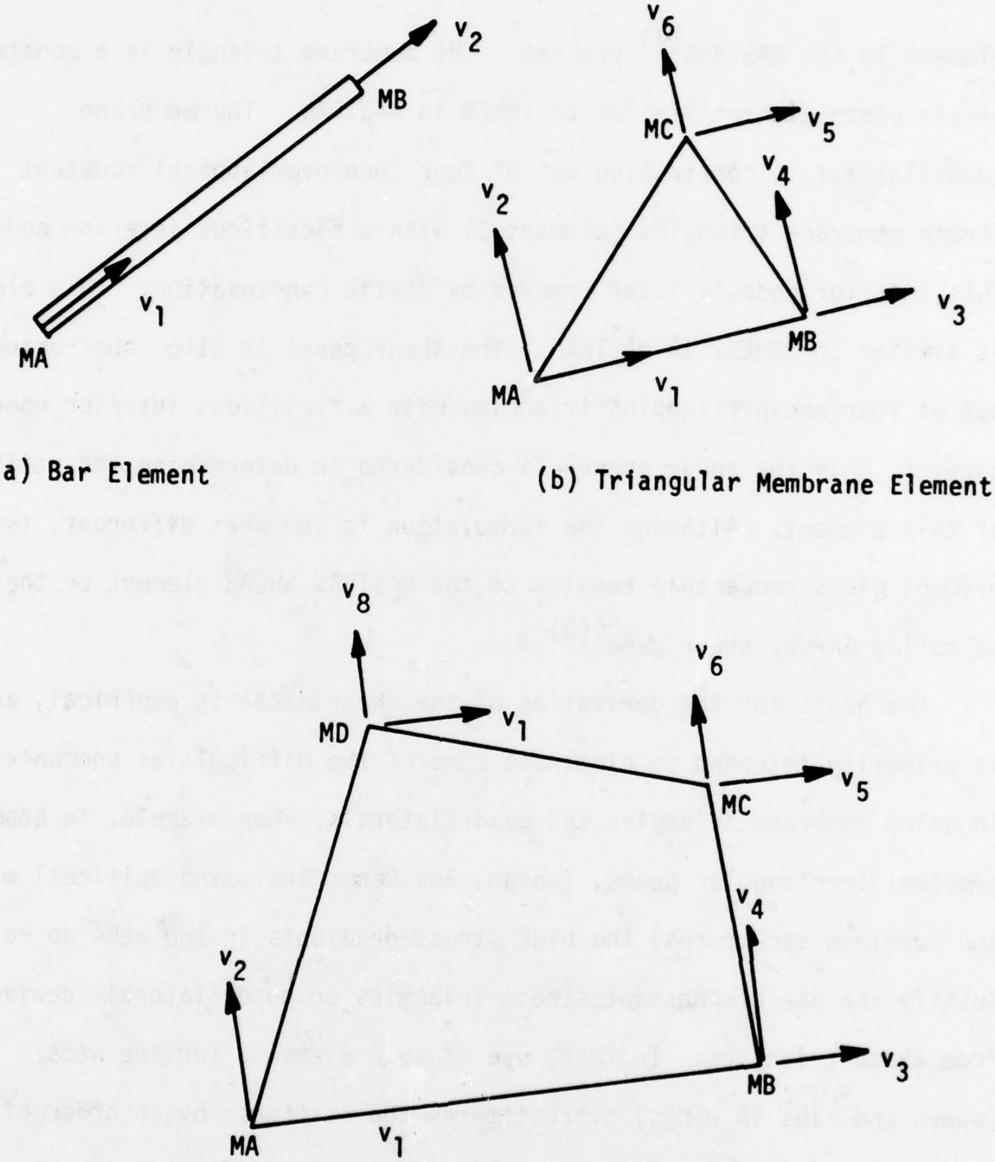


FIG. 1: Continuum and Finite Element Model



(a) Bar Element

(b) Triangular Membrane Element

(c) Quadrilateral or Shear Panel

FIG. 2: Elements and Local Coordinate System

element in the NASTRAN[7] program. The membrane triangle is a constant strain plate element similar to TRMEM in NASTRAN. The membrane quadrilateral is constructed out of four (non-overlapping) constant strain membrane triangles (element 2) with a fictitious interior node. This interior node is later removed by static condensation. This element is similar to QDMEM2 in NASTRAN. The shear panel is also constructed out of four non-overlapping triangles with a fictitious interior node. However, only the shear energy is considered in determining the stiffness of this element. Although the formulation is somewhat different, this element gives comparable results to the NASTRAN SHEAR element or the so called Garvey shear panel[8].

The basis for the derivation of the shear panel is empirical, and it is primarily intended to eliminate some of the difficulties encountered in using membrane triangles and quadrilaterals. For example, in beam problems (rectangular beams, I-beam, Box Beams including multicell wings and fuselage structures) the high stress gradients in the webs do not justify the use of constant strain triangles or quadrilaterals derived from these triangles. In fact, use of such elements for the webs (spars and ribs in wings) overestimates the stiffness by an order of magnitude. Aerospace engineers have offset this difficulty to a large extent by judicious use of membrane elements in conjunction with the shear panels. In fact the early finite element models of wings and fuselages consisted primarily of bars and shear panels. However, the present practice of using membrane triangles and quadrilaterals for the top and bottom skins, bars for the posts, spar and rib caps, and shear

panels for the spars and ribs eliminates to a large extent the need for determining the equivalent thicknesses and cross-sectional areas in the bars and shear panels model. The models consisting of these elements are most satisfactory for determining the primary load paths in built-up structures such as wings and fuselages. In addition the simplicity of these elements makes interpretation of the results easy and also keeps the analysis costs low because the stiffness matrices of these elements can be generated in a fraction of a second. The detailed formulation and additional information on these elements are given in Section 3.

In finite element analysis, a large proportion of the time is spent in the solution of the force displacement relations. The program uses standard Gaussian elimination with modifications to take into account the symmetry and sparseness characteristics of the stiffness matrix. The details of the solution scheme and storage of the stiffness matrix are given in Sections 2 and 5. "ANALYZE" is an incore program whose core requirements depend on the problem size, primarily measured in terms of the number of degrees of freedom and the size of the semi-bandwidth. However, the bandwidth per se is not considered in the program. With an available core of about $100K_8$ one can solve problems of up to 300 to 400 degrees of freedom. With the full core of a machine like the CDC 6600, it is possible to solve problems of up to 1500 degrees of freedom and a comparable number of elements. The details of core requirements are discussed in Appendix A.

The program is written in standard ANSI Fortran IV.

2. ANALYSIS

In the finite element analysis the continuum is replaced by a discrete model consisting of a finite number of nodes connected by elements (members). The rationale in such an approximation is that the response between the nodes (i.e., in the elements) can be expressed as a function of the response at the nodes. The functional relationship between the two responses is approximated by various interpolation functions or shape functions. The type of functions depends on the complexity of the problem at hand. This discretization reduces the original differential equations of the continuum to a set of algebraic equations which can be solved much more readily on digital computers.

The equations of the finite element analysis can be derived conveniently by considering the strain energy of the deformed system. For example, if the elastic body is idealized by m finite elements connecting q nodes (See Figure 1), the strain energy of the i^{th} element can be written as

$$\tau_i = \frac{1}{2} \int_{V_i} \underset{\sim}{\sigma}_i^{t*} \; \underset{\sim}{\varepsilon}_i \, dV \qquad (1)$$

where $\underset{\sim}{\sigma}_i$ and $\underset{\sim}{\varepsilon}_i$ are the stress and strain vectors and V_i is the volume of the element. For a linearly elastic body the relation between stress and strain can be written as

$$\underset{\sim}{\sigma}_i = \underset{\sim}{E}_i \; \underset{\sim}{\varepsilon}_i \qquad (2)$$

---

* Superscript t on a matrix represents transpose

where $\underset{\sim}{E}_i$ is the symmetric matrix of material elastic constants. For typical plane stress problems the elastic constants matrix is of dimension 3x3. For an isotropic material in plane stress problems the elements of $\underset{\sim}{E}$ are as follows:

$$\underset{\sim}{E} = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\mu) \end{bmatrix} \tag{3}$$

where E and μ are the elastic modulus and poisson's ratio of the material respectively. For an orthotropic material the elastic constants matrix is given by

$$\underset{\sim}{E} = \frac{E_1}{1-\beta\mu^2} \begin{bmatrix} 1 & \mu\beta & 0 \\ \mu\beta & \beta & 0 \\ 0 & 0 & \frac{G}{E_1}(1-\beta\mu^2) \end{bmatrix} \tag{4}$$

where E_1 and E_2 are the longitudinal and transverse moduli, respectively, in the directions of the material property axes. β is the ratio of transverse to longitudinal modulus (E_2/E_1). G and μ are the shear modulus and poisson's ratio respectively.

The essence of the finite element approximation is that the internal displacements of the elements are expressed as functions of the displacements of the discrete nodes to which they are connected. The local coordinate systems and the nodal degrees of freedom of the four elements are shown in Figure 2. The functional relationship between the element internal displacements and the discrete nodal displacements is given by

$$\underset{\sim}{w}_i = \underset{\sim}{\phi}_i \underset{\sim}{v}_i \tag{5}$$

where the matrix $\underset{\sim}{w}_i$ represents the displacements in the element which are functions of the spatial coordinates (x, y). The shape function $\underset{\sim}{\phi}_i$ is a rectangular matrix, and its elements are also functions of the spatial coordinates. The vector $\underset{\sim}{v}_i$ represents the nodal displacements in the direction of the element degrees of freedom in the local coordinate system (Figure 2). Now the strain-displacement relations can be written as

$$\underset{\sim}{\varepsilon}_i = \underset{\sim}{B}\,\underset{\sim}{w}_i \tag{6}$$

where $\underset{\sim}{B}$ is a differential operator. For a plane stress problem $\underset{\sim}{B}$ is given by

$$\underset{\sim}{B} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \tag{7}$$

Substitution of Equations 2, 5 and 6 in 1 gives the expression for strain energy in the following form

$$\tau_i = \frac{1}{2}\,\underset{\sim}{v}_i^t\,\underset{\sim}{k}_i\,\underset{\sim}{v}_i \tag{8}$$

where $\underset{\sim}{k}_i$ is the element (member) stiffness matrix with respect to the discrete coordinates $\underset{\sim}{v}$ and is given by

$$\underset{\sim}{k}_i = \int_{V_i} \underset{\sim}{\phi}_i^t\,\underset{\sim}{B}^t\,\underset{\sim}{E}_i\,\underset{\sim}{B}\,\underset{\sim}{\phi}_i\,dV \tag{9}$$

An alternate but a convenient method of determining the elements of the member stiffness matrix is by invoking the principle of virtual work[9]

which gives

$$1 \cdot k_{pq} = \int_{V_i} \underset{\sim}{\sigma}_i^{(p)^t} \; \underset{\sim}{\varepsilon}_i^{(q)} \, dV \tag{10}$$

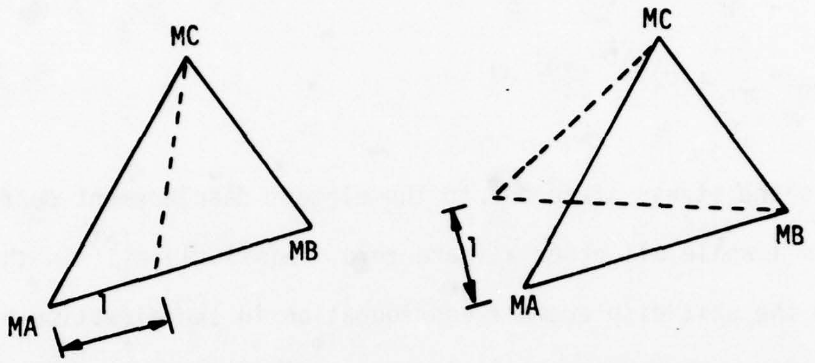
where $\underset{\sim}{\sigma}_i^{(p)}$ is the stress state due to the element displacement configuration in which $v_p = 1$ while all other v's are zero. Similarly $\underset{\sim}{\varepsilon}_i^{(q)}$ is the strain state due to the unit displacement configuration in the direction of the q^{th} degree of freedom. These two conditions are shown in Figure 3 for the degrees of freedom 1 and 2 of the membrane triangle. It should be noted that besides assuming appropriate shape functions, the integration in Equations 9 or 10 is one of the difficult tasks in the case of complex elements in finite element analysis. However, for membrane elements this integration does not present any difficulties as will be seen in the next section. For more complex elements the usual practice is to adopt numerical integration schemes.[10,11]

From Equation 8 and Castigliano's first theorem, the relation between the element nodal forces and the displacements may be written as

$$\underset{\sim}{s}_i = \left[\frac{\partial \tau_i}{\partial v_j} \right] = \underset{\sim}{k}_i \, \underset{\sim}{v}_i \tag{11}$$

where $\underset{\sim}{s}_i$ is the element nodal force matrix corresponding to the displacement matrix $\underset{\sim}{v}_i$. Similar force-displacement relations for the total structure can be derived from the strain energy of the structure. The total strain energy Γ of the structure can be written as the sum of the energies of the individual components.

$$\Gamma = \sum_{i=1}^{m} \tau_i = \frac{1}{2} \sum_{i=1}^{m} \underset{\sim}{v}_i^t \, \underset{\sim}{k}_i \, \underset{\sim}{v}_i \tag{12}$$



(a) First Unit Mode (b) Second Unit Mode

FIG. 3: Examples of Unit Displacement Modes

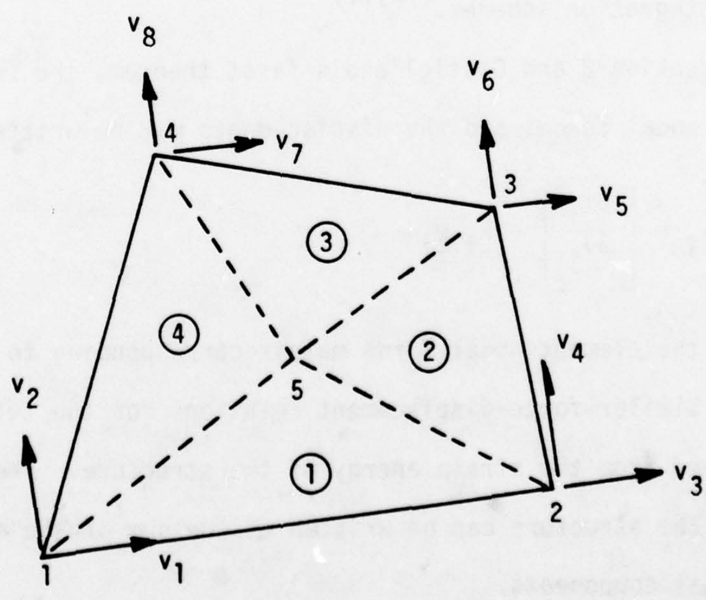


FIG. 4: Quadrilateral or Shear Panel
Divided into Four Triangles

In general, for most structures, it is convenient to define a local coordinate system for each element and a global coordinate system for the total structure. In such a case the element and structure generalized coordinates can be related by

$$\underset{\sim}{v}_i = \underset{\sim}{a}_i \underset{\sim}{u} \tag{13}$$

where $\underset{\sim}{a}_i$ is the compatibility matrix. Its elements can be determined by kinematic reasoning alone provided the structure is kinematically determinate. The matrix $\underset{\sim}{u}$ is the generalized displacement vector of the structure in the global coordinate system. It is interesting to note that Equation 13 not only transforms element displacements from local to global coordinates but also gives information about how the elements are connected to the structure. From Equation 13 and the principle of virtual work it is easy to show that the transformation between the forces on the structure and the element internal forces is given by

$$\underset{\sim}{P} = \underset{\sim}{a}_i^t \underset{\sim}{s}_i \tag{14}$$

where $\underset{\sim}{P}$ is the force vector on the structure in the global coordinate system. The transformation given in Equation 14 is sometimes referred to as a contragradient transformation.[12]

Substitution of Equation 13 in 12 gives the expression for the total strain energy in the form

$$\Gamma = \frac{1}{2} \underset{\sim}{u}^t \underset{\sim}{K} \underset{\sim}{u} \tag{15}$$

where $\underset{\sim}{K}$, the total stiffness matrix of the structure, is written as the sum of the component stiffness matrices.

$$\underset{\sim}{K} = \sum_{i=1}^{m} \underset{\sim}{a}_i^t \underset{\sim}{k}_i \underset{\sim}{a}_i \tag{16}$$

Again using Castigliano's first theorem the relation between the generalized force matrix $\underset{\sim}{P}$ corresponding to the displacement matrix $\underset{\sim}{u}$ may be written as

$$\underset{\sim}{P} = \left[\frac{\partial \Gamma}{\partial u_j} \right] = \underset{\sim}{K}\,\underset{\sim}{u} \tag{17}$$

In most structural analysis problems the stiffness matrix $\underset{\sim}{K}$ is sparsely populated. It is essential to take advantage of this fact in solving the load deflection equations (Equation 17), particularly in the case of problems with a large number of degrees of freedom where the cost of computation can be prohibitive otherwise. The "ANALYZE" program uses Gaussian elimination with modifications to take into account the symmetry and sparseness of the stiffness matrix.

Basically Gaussian elimination involves decomposition of the stiffness matrix by

$$\underset{\sim}{K} = \underset{\sim}{L}\,\underset{\sim}{D}\,\underset{\sim}{L}^t \tag{18}$$

where $\underset{\sim}{L}$ is the unit lower triangular matrix and $\underset{\sim}{D}$ is a diagonal matrix. The advantage of this decomposition scheme is that the $\underset{\sim}{L}$ matrix retains some of the sparseness characteristics of $\underset{\sim}{K}$ which consequently reduces the numbers of computations. Also $\underset{\sim}{L}$ and $\underset{\sim}{D}$ can be assigned the same storage as $\underset{\sim}{K}$.

The next step is the forward substitution by

$$\underset{\sim}{L}\,\underset{\sim}{Y} = \underset{\sim}{P} \tag{19}$$

where the matrix $\underset{\sim}{Y}$ is given by

$$\underset{\sim}{Y} = \underset{\sim}{D}\,\underset{\sim}{L}^t\,\underset{\sim}{u} \tag{20}$$

In Equation 19 the solution of $\underset{\sim}{Y}$ can be accomplished by simple forward substitution. Once $\underset{\sim}{Y}$ is obtained, $\underset{\sim}{u}$ can be solved by back substitution

using Equation 20. The last two steps together are generally referred to as Forward-Back Substitution (FBS). Solution of Equation 17 for multiple load vectors involves the decomposition of the stiffness matrix once and repetition of FBS as many times as there are load vectors.

With the help of these basic equations the steps in the finite element analysis can be outlined as follows:

1. Input information consists of
   a. Geometry of the structure
      Node Coordinates
      Element Connections
      Section Properties
   b. Material properties
   c. Boundary conditions
   d. Loading
   e. Clues for appropriate (desired) output.
2. Element information consists of
   a. Determination of the local coordinate system for each element.
   b. Selection of the appropriate shape functions (Equation 5).
   c. Determination of the element stiffness matrix (Equation 9 or 10).
3. Transformation of the element stiffness matrix to the global coordinate system (Equation 16 without summation).
4. Determination of the structure stiffness matrix by summation of the component stiffnesses (Summation in Equation 16).
5. Incorporation of the boundary conditions.
6. Solution of the load-deflection equations (Equations 17, 18, 19, and 20).

7. Determination of the element displacements in their local coordinate system (Equation 13).

8. Determination of the stresses in each element (Equations 6, 5, and 2).

9. Output the structure displacements, element stresses and other information such as element strain energies, etc.

The next section consists of the details of the stiffness matrix formulations for the four elements in this program.

3. FINITE ELEMENTS

The program "ANALYZE" has four elements as mentioned earlier. They are all membrane elements. These four elements are generally adequate for determining the primary load paths of most aircraft structures. However, for a detailed stress analysis of local areas, higher order elements may be necessary.

BAR (ROD) ELEMENT

Basically this element is an axial force member. Its primary use is in two and three dimensional truss structures. It is also used extensively as spar and rib caps, posts around shear panels, stiffners and other line elements in aircraft structures. The local coordinate system of this element is shown in Figure 2. The positive x-axis is directed along the line joining the two ends. v_1 and v_2 represent the element end displacements. The corresponding two end forces are s_1 and s_2. The displacement field in the element is assumed to be linear, which gives constant strain. For a linearly elastic material this assumption yields constant stress as well.

If w, the displacement at any point along the length of the bar, is given by

$$w = ax + b \tag{17}$$

where a and b are two undetermined coefficients and x is the coordinate of the point in the local coordinate system, then the end displacements v_1 and v_2 are given by

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{18}$$

where x_1 and x_2 are the coordinates of the two ends in the local coordinate system. Then the shape function (Equation 5) corresponding to this linear displacement field can be written as

$$\underset{\sim}{\phi} = \frac{1}{(x_1 - x_2)} \left[(x - x_2), \ -(x - x_1) \right] \tag{19}$$

From the strain-displacement relations, the axial strain in the element is given by

$$\varepsilon_x = \frac{\partial w}{\partial x} = a \tag{20}$$

From the principle of virtual work (Equation 10) the individual elements of the member stiffness matrix can be written as

$$k_{ij} = \int_V \sigma_x^{(i)} \varepsilon_x^{(j)} \, dV = (-1)^{i+j} \frac{AE}{L} \tag{21}$$

where A is the cross-sectional area, L is the length of the member, and E is the modulus of elasticity of the material. The member stiffness matrix is given by

$$\underset{\sim}{k} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{22}$$

The member force matrix is given by

$$\underset{\sim}{s} = \underset{\sim}{k} \, \underset{\sim}{v} \tag{23}$$

The stress in the member is given by

$$\sigma_x = E \, \varepsilon_x \tag{24}$$

or

$$\sigma = \frac{s_1}{A} = \frac{-s_2}{A} \tag{25}$$

The strain energy in the element is given by

$$\tau_i = \frac{1}{2} \underset{\sim}{s}^t \underset{\sim}{v} \tag{26}$$

or

$$\tau_i = \frac{1}{2} \sigma_x \varepsilon_x A L \tag{27}$$

TRIANGULAR MEMBRANE ELEMENT

The membrane triangle is the basic plate element in the program. It is used to construct the membrane quadrilateral as well as the shear panel with some modifications. The membrane triangle can be used effectively in all cases where the primary loading is inplane forces. These include top and bottom skins of aircraft wings, flanges of I and box beams when they are subjected to constant normal stresses (tension or compression) only and skins of sandwich construction. However, they are not suitable for situations where high stress gradients exist. For example, they are unsuitable for spars and ribs of wings and other lifting surfaces, webs of I and box beams and flat plates where the primary load is bending. If used in such cases, they overestimate the stiffness or generate singularity. Figure 2 shows the triangle elements with the local coordinate system. The generalized coordinates v_1, v_2, ---, v_6 represent the inplane displacements of the three nodes in the local coordinate system. The displacement field in the element is assumed to be linear. This gives constant strain in the element. For a linearly elastic material the stress in the element will also be constant.

The linear displacement field in the element can be represented by

$$w_x = a_1 x + b_1 y + c_1$$
$$w_y = a_2 x + b_2 y + c_2 \tag{28}$$

where w_x and w_y are the x-y displacements in the plane of the plate in the local coordinate system. a_1, b_1 etc. are the six undetermined coefficients. Equation 28 can be written in matrix form as follows:

$$\underset{\sim}{w} = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{bmatrix} \tag{29}$$

The six unknown coefficients can be uniquely determined by the six boundary conditions at the nodes.

$$\begin{bmatrix} v_1 \\ v_3 \\ v_5 \\ \hline v_2 \\ v_4 \\ v_6 \end{bmatrix} = \left[\begin{array}{ccc:ccc} x_1 & y_1 & 1 & 0 & 0 & 0 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ x_3 & y_3 & 1 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & x_1 & y_1 & 1 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ 0 & 0 & 0 & x_3 & y_3 & 1 \end{array}\right] \begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ \hline a_2 \\ b_2 \\ c_2 \end{bmatrix} \tag{30}$$

where x_1, y_1, ----, x_3 and y_3 are the coordinates of the three nodes of the triangle in the local coordinate system. It should be noted that the

nodal displacements are grouped into x and y directions, so that the nodal coordinate matrix on the right hand side partitions into a diagonal matrix. The inversion of the partitioned diagonal matrix involves simply the inversion of the component matrix. Now the shape matrix $\underset{\sim}{\phi}$ is given by

$$\underset{\sim}{\phi} = \underset{\sim}{x}\,\underset{\sim}{Z}^{-1} \tag{31}$$

where the matrix $\underset{\sim}{x}$ is given by

$$\underset{\sim}{x} = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \tag{32}$$

and the $\underset{\sim}{Z}$ matrix is given by

$$\underset{\sim}{Z} = \left[\begin{array}{c|c} \underset{\sim}{X} & 0 \\ \hline 0 & \underset{\sim}{X} \end{array}\right] \tag{33}$$

The coordinate matrix $\underset{\sim}{X}$ is given by

$$\underset{\sim}{X} = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \tag{34}$$

It is interesting to note that each column of $\underset{\sim}{Z}^{-1}$ represents a unit displacement mode: i.e. the j^{th} column of the inverse represents a displacement mode in which $v_j = 1$ while all other nodal displacements are zero (See Figure 3). This fact is used to advantage in determining the elements of the member stiffness matrix.

From linear strain-displacement relations the strains can be written as

$$\varepsilon_x = \frac{\partial w_x}{\partial x} = a_1 \tag{35}$$

$$\varepsilon_y = \frac{\partial w_y}{\partial y} = b_2 \tag{36}$$

$$\varepsilon_{xy} = \frac{\partial w_x}{\partial y} + \frac{\partial w_y}{\partial x} = b_1 + a_2 \tag{37}$$

From the principle of virtual work (Equation 10) the elements of the member stiffness matrix can be written as

$$k_{ij} = \int_V \underset{\sim}{\sigma}^{(i)^t} \underset{\sim}{\varepsilon}^{(j)} \, dV = \int_V \underset{\sim}{\varepsilon}^{(i)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(j)} \, dV \tag{38}$$

where $\underset{\sim}{\sigma}^{(i)}$ and $\underset{\sim}{\varepsilon}^{(j)}$ are the stress and strain matrices corresponding to the unit displacement modes explained under Equation 34. Since the linear displacement variation implies constant strain, the integral in Equation 38 can be replaced by the volume of the element:

$$k_{ij} = \frac{1}{2} |\underset{\sim}{X}| \; t \; \underset{\sim}{\varepsilon}^{(i)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(j)} \tag{39}$$

where $|\underset{\sim}{X}|$ is the determinant of the nodal coordinate matrix which represents twice the area of the element and t is the thickness of the element. Now the stiffness matrix of the element is given by

$$\underset{\sim}{k} = \frac{1}{2} |\underset{\sim}{X}| \; t \begin{bmatrix} \underset{\sim}{\varepsilon}^{(1)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(1)} & \underset{\sim}{\varepsilon}^{(1)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(2)} & \cdots & \underset{\sim}{\varepsilon}^{(1)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(6)} \\ \underset{\sim}{\varepsilon}^{(2)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(1)} & \underset{\sim}{\varepsilon}^{(2)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(2)} & \cdots & \underset{\sim}{\varepsilon}^{(2)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(6)} \\ \vdots & \vdots & & \vdots \\ \underset{\sim}{\varepsilon}^{(6)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(1)} & \underset{\sim}{\varepsilon}^{(6)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(2)} & \cdots & \underset{\sim}{\varepsilon}^{(6)^t} \underset{\sim}{E} \, \underset{\sim}{\varepsilon}^{(6)} \end{bmatrix} \tag{40}$$

The member force matrix is given by

$$\underset{\sim}{s} = \underset{\sim}{k} \, \underset{\sim}{v} \tag{41}$$

The stress matrix in the element is given by

$$\underset{\sim}{\sigma} = \underset{\sim}{E} \, \underset{\sim}{\varepsilon} \tag{42}$$

The strain energy in the element is given by

$$\tau_i = \frac{1}{2} \underset{\sim}{S}^t \underset{\sim}{v} \tag{43}$$

or

$$\tau_i = \frac{1}{4} |\underset{\sim}{X}| \, t \, \underset{\sim}{\sigma}^t \underset{\sim}{\varepsilon} \tag{44}$$

The next important step in the evaluation of the stress state in an element is the selection of a suitable failure criteria because of the combined stresses (σ_x, σ_y, and σ_{xy}) in plate elements. The modified energy of distortion or Von-Mises criteria is adopted to determine the effective stress in an element. The effective stress is given by

$$\sigma_{eff} = (\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\sigma_{xy}^2)^{1/2} \tag{45}$$

The margin of safety is evaluated by first determining the effective stress ratio (ESR)

$$ESR = \left[\left(\frac{\sigma_x}{XX}\right)^2 + \left(\frac{\sigma_y}{YY}\right)^2 - \left(\frac{\sigma_x \sigma_y}{XXYY}\right) + \left(\frac{\sigma_{xy}}{ZZ}\right)^2 \right]^{1/2} \tag{46}$$

where XX and YY are the tension or compression allowable in the x and y directions, respectively, and ZZ is the shear allowable. Then the margin of safety (MS) is determined by

$$MS = \frac{1-ESR}{ESR} \tag{47}$$

If the user does not provide the allowable stress values, then default values of 60,000 psi for tension and compression allowables in both directions and 36,000 psi for the shear allowable are used.

QUADRILATERAL MEMBRANE ELEMENT

The quadrilateral element is most frequently used to represent membrane skins unless the corners etc. require the use of the triangular element.

Figure 4 shows the local coordinate system and the generalized coordinates (displacements) v_1 through v_8. The element is assumed to be a flat plate, and all nodes are assumed to lie on a plane connecting the first three nodes (1, 2, and 3). In effect the warping in the element is ignored. This approximation results in an overestimation of the stiffness of a truly warped quadrilateral element. In most cases the effect of the approximation is small, and it can be further reduced by reducing the mesh size of the model in the regions of high warping. However, if the warp is too large, the quadrilateral should be broken up into two or more triangles.

As mentioned earlier, the stiffness of the quadrilateral element is determined by breaking it into four component triangles as shown in Figure 4. A fictitious node in the quadrilateral is located by averaging the coordinates of the four nodes as given by

$$x_5 = \frac{x_1 + x_2 + x_3 + x_4}{4} \tag{48}$$

$$y_5 = \frac{y_1 + y_2 + y_3 + y_4}{4} \tag{49}$$

The stiffness of the four triangles is then computed by Equation 40 in the local coordinate system shown in Figure 2c. Addition of the four stiffness matrices gives a 10 x 10 stiffness matrix with two degrees of freedom included for the fifth node. This fictitious node is later removed by static condensation before adding to the total structure. The procedure for static condensation is outlined next.

The force displacement relations of the 5 node quadrilateral are written as

$$\underset{\sim}{R}_Q = \underset{\sim}{k}_Q \, \underset{\sim}{r}_Q \tag{50}$$

where the subscript refers to the quadrilateral element with 5 nodes. Equation 50, partitioned to isolate the degrees of freedom of the fifth node, can be written as

$$\left[\begin{array}{c} \underset{\sim}{R}_I \\ \hline \underset{\sim}{R}_{II} \end{array}\right] = \left[\begin{array}{c|c} \underset{\sim}{K}_{I,\,I} & \underset{\sim}{K}_{I,\,II} \\ \hline \underset{\sim}{K}_{II,\,I} & \underset{\sim}{K}_{II,\,II} \end{array}\right] \left[\begin{array}{c} \underset{\sim}{r}_I \\ \hline \underset{\sim}{r}_{II} \end{array}\right] \tag{51}$$

Equation 51 can be written as two separate equations

$$\underset{\sim}{R}_I = \underset{\sim}{k}_{I,I} \, \underset{\sim}{r}_I + \underset{\sim}{k}_{I,II} \, \underset{\sim}{r}_{II} \tag{52}$$

$$\underset{\sim}{R}_{II} = \underset{\sim}{k}_{II,I} \, \underset{\sim}{r}_I + \underset{\sim}{k}_{II,II} \, \underset{\sim}{r}_{II} \tag{53}$$

Since the fifth node does not actually exist in the original model, no external forces can be applied to this node. This condition gives

$$\underset{\sim}{r}_{II} = -\underset{\sim}{k}^{-1}_{II,II} \, \underset{\sim}{k}_{II,I} \, \underset{\sim}{r}_I \tag{54}$$

Substitution of Equation 54 in 52 gives

$$\underset{\sim}{R}_I = (\underset{\sim}{k}_{I,I} - \underset{\sim}{k}_{I,II} \, \underset{\sim}{k}^{-1}_{II,II} \, \underset{\sim}{k}_{II,I}) \, \underset{\sim}{r}_I \tag{55}$$

From Equation 55 the stiffness matrix of the original quadrilateral can be written as

$$\underset{\sim}{k} = \underset{\sim}{k}_{II} - \underset{\sim}{k}_{I,II} \, \underset{\sim}{k}^{-1}_{II,II} \, \underset{\sim}{k}_{II,I} \tag{56}$$

The stiffness as obtained by Equation 56 is added to the total structure after appropriate coordinate transformations to the global coordinate system.

When the structure displacements are determined, the fifth node displacements can be determined by Equation 54. Now the stresses in each triangle can be determined as before. The effective stress ratio is determined for each triangle separately (Equation 46), and then a weighted average is used in computing the effective stress ratio and the margin of safety. This weighted average is computed by

$$ESR = \frac{(ESR)_1\ \Delta_1 + (ESR)_2\ \Delta_2 + (ESR)_3\ \Delta_3 + (ESR)_4\ \Delta_4}{\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4} \tag{57}$$

where $(ESR)_1$ thru $(ESR)_4$ are the effective stress ratios of the four triangles. Δ_1 thru Δ_4 are the respective planform areas of the triangles. Now the margin of safety MS is computed as before by Eq. 47.

SHEAR PANEL

As the name indicates, the shear panel is devised for the purpose of representing shear transmitting elements. For example in wing structures the top and bottom skins can be represented by membrane (triangle and quadrilateral) elements. If the same elements are used for spars and ribs, the resulting finite element model grossly overestimates the stiffness of the structure. What this means is that the displacements obtained by this model will be much smaller, or if this model is used for dynamic analysis, the frequencies of the structure will be much higher and cannot be matched with the results obtained from ground vibration tests. This behavior is due to the assumption of constant strain (stress) in the membrane element formulations. Most web elements in box or I-beams carry primarily shear and some normal stresses. In other words their deformation is primarily due to shear and not due to normal stresses. The normal stresses in webs usually have steep stress gradients, and the

assumption of constant stress (or strain) is not justified. To offset this difficulty, and yet preserve the simplicity of the constant strain elements, a shear panel was formulated (Reference 8) with the assumption that it carries only shear stresses. The bars and other membrane elements that surround the shear panel are supposed to carry the normal stresses. Such a situation does not actually exist in reality, and thus the shear panel is an empirical element. However, the models built on such an assumption appear to produce satisfactory results.

Until recently it was a common practice in aircraft companies to model wings, fuselages, and empennage structures simply by bars and shear panels to obtain primary load path information. In such idealizations it was a common practice to assign a third of the cross-sectional area as spar and rib caps and the remainder for the shear panels. It should be pointed out that every shear panel must be surrounded on all four sides by normal stress carrying elements such as bars or membrane or bending elements. If the natural model does not contain such an element on any side of the shear panel, a nominal (or fictitious) bar (post) must be provided. Otherwise the model will have a singularity.

The shear panel in "ANALYZE" is constructed out of four triangles with the fictitious node inside as in the membrane quadrilateral discussed earlier. However, the stiffness matrices of the component triangles are determined by considering only the shear strain energy (Equation 39).

$$k_{ij} = \frac{1}{2} |\underset{\sim}{X}| \; t \; \varepsilon_{xy}^{(i)} \; G \; \varepsilon_{xy}^{(j)} \tag{58}$$

where G is the shear modulus, and $\varepsilon_{xy}^{(i)}$ and $\varepsilon_{xy}^{(j)}$ are the shear strains due to the unit displacement modes discussed earlier. There is one point that must be made here. The shear stress (strain) in an element changes with the orientation of the reference axis. Thus the stiffness matrix of the element can be sensitive to the reference axis. For rectangular elements the shear strain energy would be the same regardless of which side is selected for the reference axis. However, for quadrilaterals the stiffness matrix does depend on the reference axis. The errors produced by such departures are usually not significant, but it is worthwhile to make note of the assumptions involved. The ANALYZE program has a provision for specifying any one of the four sides of the quadrilateral as the reference axis.

As in the quadrilateral element the shear stresses in all four triangles are determined separately but with respect to the same reference axis. Of course, the normal stresses in the shear panels have no meaning. The margin of safety is determined by a weighted average of the effective stress ratios(ESR) as in the quadrilateral. The strain energy is determined by considering only the shear stress and strain.

4. ORGANIZATION OF THE PROGRAM

The material presented in this section is intended either to help introduce changes into the program or to expand its scope for the specific needs of a researcher as the authors have done in the past ten years. The steps outlined at the end of Section 2 are summarized in the flow-chart in Figure 5. There are a total of 16 boxes in the flow-chart. Each of these boxes generally involves one or more subroutines. The subroutines that belong to each of these boxes are identified first, then the function of each subroutine will be discussed in the next section with the help of the equations given in Sections 2 and 3.

Box 1 - Input

Input in the present version of the "ANALYZE" program is not in subroutine form. However, the input statements are all at the beginning of the program, and thus they can be grouped into a single subroutine. Alternatively, one can generate an input routine of his own with provisions like one card per each element and a card for each node etc. For example, it is relatively easy to write a subroutine with NASTRAN type input. The description of the various arrays (See input instructions) and their dimension requirements given in Appendix A can be quite helpful in writing such an input routine.

Box 2 - Map Stiffness Matrix

This step involves a single subroutine called "POP". The purpose of this routine is simply to estimate the storage requirements of the stiffness matrix and to map its profile. The stiffness matrix is stored in a single array called SK. The elements of the matrix are stored columnwise

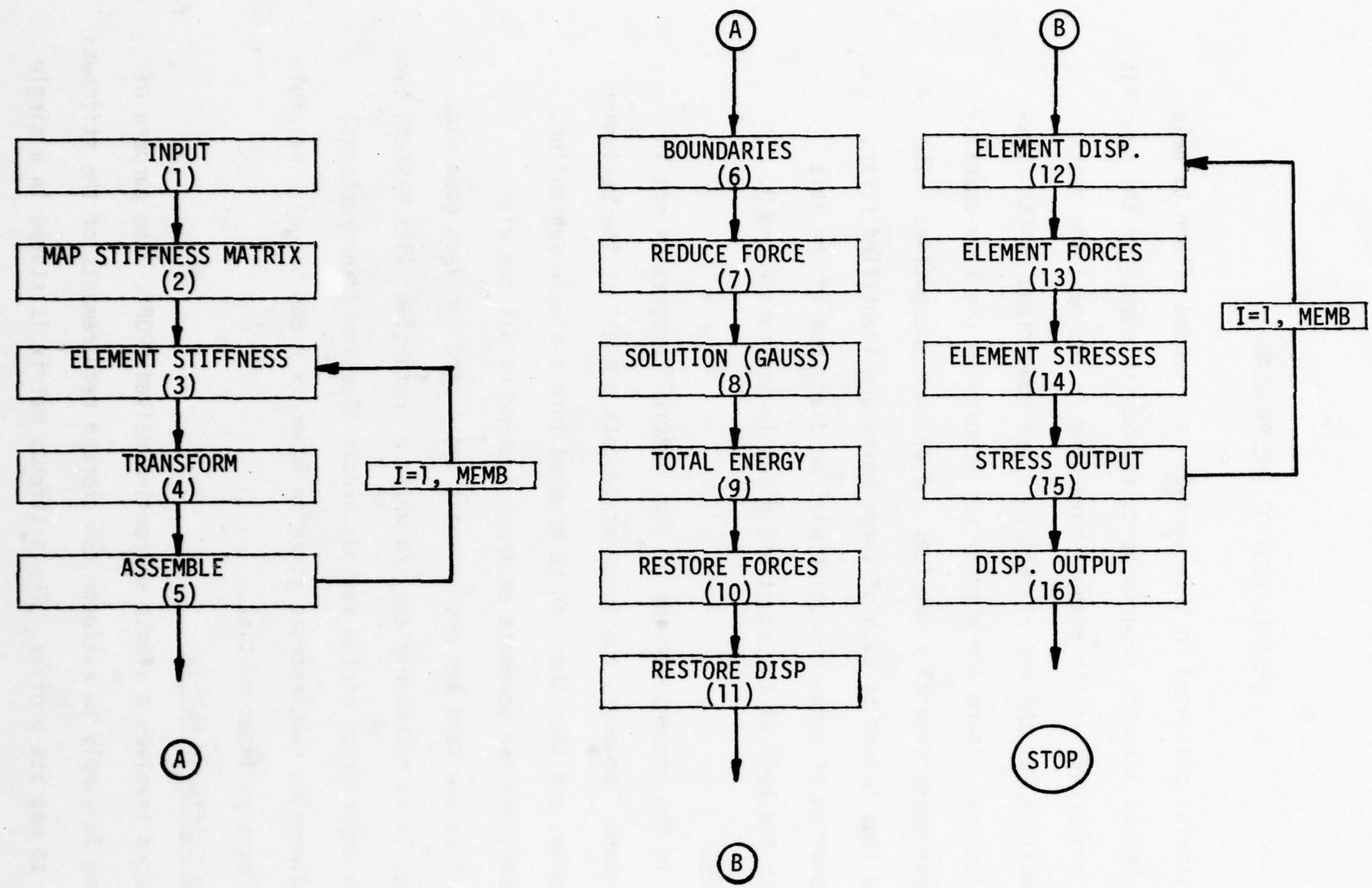


Figure 5. Flow Chart for Program "ANALYZE"

starting from the first non-zero element in the column to the diagonal element. Since the matrix is symmetric, only the upper triangle is stored.

Box 3 - Element Stiffness

There are four elements in the program. All of them require the subroutine "COORD". In addition all the plate elements require the routine "ELSTIC". The remaining subroutines are listed separately for each element.

i. Bar (Rod) Element:

The bar element is shown in Figure 2a with the local coordinate system and degrees of freedom. This element requires the subroutine "ELSTIF" which generates the bar stiffness matrix in the local coordinate system and also transforms it to the global coordinate system.

ii. Triangular Membrane Element:

The element and its local coordinate system are shown in Figure 2b. The subroutine "PLSTIF" is the only other routine required by this element. It generates the stiffness matrix of the triangle in the local coordinate system.

iii. Quadrilateral Membrane Element and Shear Panel

The elements and their local coordinate system are shown in Figure 2c. The subroutines "QDRLTL", "PLSTIF", "SUM", "CONDNS", "CHANGE" and "CRAMER" are the additional routines required by these elements. Together these subroutines generate the stiffness matrix of either the quadrilateral membrane or shear panel. The routine "QDRLTL" calls "PLSTIF", "SUM" and "CONDNS". The routine "PLSTIF" calls "CRAMER". Similarly "CONDNS" calls "CHANGE".

Box 4 - Transform

This step involves a single subroutine called "TRNSFM". It transforms the stiffness matrices of the triangles, quadrilaterals, and shear panels from the local to the global coordinate system.

Box 5 - Assemble

"ASEMBL" is the only subroutine used in this step. Its purpose is to add the element stiffness matrices to the total stiffness matrix of the structure. The steps 3 thru 5 form a loop in which all the element stiffness matrices are computed and assembled into the total stiffness matrix.

Box 6 - Boundaries

The routine called "BOUND2" eliminates the rows and columns of the stiffness matrix corresponding to the support degrees of freedom of the structure. In addition it also condenses the stiffness matrix.

Box 7 - Reduce Force

This step involves a routine called "REDUCE". It eliminates the rows of the force matrix corresponding to the support degrees of freedom.

Box 8 - Solution of the Force Deflection Equations

The routine "GAUSS" solves the load deflection equations by Gaussian elimination. A large percentage of the analysis time (80 to 90%) is spent in this routine, and its efficiency is extremely important in reducing the costs of the analysis. At the end of this step the displacements of the structure are available in condensed form (excluding boundary degrees of freedom) in the global coordinate system.

Box 9 - Total Energy

The total energy of the structure is computed by

$$W = \frac{1}{2} \underset{\sim}{R}^t \underset{\sim}{r}$$

The strain energy of the structure (U) is computed by adding the strain energies of the elements in step 14 (Box 14). A comparison of W and U provides an equilibrium check.

Boxes 10 and 11 - Restore Forces and Displacements

These two boxes use the same routine called "RESTOR". The purpose of this routine is to restore the force and deflection matrices to their original dimension to include the boundary degrees of freedom. Its purpose is essentially opposite to that of the routine "REDUCE" in Box 7.

Box 12 - Element Displacements

The routine "COORD" and "ELFORC" facilitate extraction and transformation of the element displacements from the global to the local coordinate system.

Box 13 - Element Forces

This step is not in all versions of "ANALYZE". Element forces are not necessary to compute stresses. However, this step can be restored if the element shear flows and other force information are necessary.

Box 14 - Element Stresses

The details of this step depend on the type of element.

i. Bar (Rod) Element:

The stress in this element is computed in the program itself. No additional routines are involved. At the same time the element strain energy is also computed.

ii. Triangular Membrane Element:

The subroutines "STRESS" and "CRAMER" are involved in this step. The routine "STRESS" calls "CRAMER". The purpose of this routine is to calculate stresses in the triangular element. In addition this routine calculates strain energy and the effective stress in the element (See Equations 44 and 45).

iii. Quadrilateral Membrane and Shear Panel

This step involves routines "ELSTIC", "QDRLTL", "PLSTIF", "SUM", "CONDNS", "CRAMER", "QLSTRS" and "STRESS". It should be noted that the routine "QDRLTL" calls "PLSTIF", "SUM and "CONDNS". "PLSTIF" in turn calls "CRAMER".

Box 15 - Stress Output

The instructions for the output of the table of stresses are in the main program. No subroutine is used for the output itself. The steps 12 thru 15 form a loop in which the stress information for all the elements is computed and printed in a table. This is one of the two main tables of output of this program. Explanation of this table is given in the section on output (Section 7).

Box 16 - Displacement Output

This step involves a single subroutine called "PRNTDR". This routine prints out the second important table of output which contains information about the nodes. This information includes the coordinates of the nodes, applied forces and the calculated displacements for each node. The detailed explanation of this table is given in the section on output (Section 7).

In addition to the above 16 steps there are instructions for weight computations and other details, and their purpose can be identified from the program. There are very few comment cards in the main body of the program and this omission is by design in order to avoid continuous updating. The user can incorporate his own comment cards with the help of the explanation given in this section.

5. DESCRIPTION OF THE SUBROUTINES

"ANALYZE" consists of the main program and 21 Subroutines. The main program has 260 cards. The length of the Subroutines varies from 15 to 62 cards. The total length of the program is under 1000 cards. A list of the Subroutines, the number of Cards in each Subroutine and other details are given in Table 1. The flow chart, Fig. 5, and the explanation in the previous section give details of the main program. The description of the Subroutines is given in the remainder of this section.

Subroutine "POP"

The purpose of Subroutine "POP" is to estimate the storage requirements of the stiffness matrix before actually determining it. This information can be generated from the element connections with the nodes. For example, if an element connects 4 nodes, and if each node has 3 degrees of freedom in the global coordinate system, then the stiffness matrix of the element would be of dimension 12 x 12. This matrix can be partitioned four ways, in both row and column directions as shown in Fig. 6. The location of these sixteen submatrices in the total stiffness matrix can be determined by the address of the nodes to which the element is connected. If the element is connected to the nodes MA, MB, MC, and MD, then the addresses of the element submatrices in the total stiffness matrix are shown in Fig. 6.

If all the elements are connected to all the nodes, then the stiffness matrix of the structure will be fully populated. The non-zero elements in the matrix are considered as population. Since most of the elements connect only a few nodes, the stiffness matrices are usually sparsely populated. Determining the profile of the stiffness matrix population is the essential function of the routine "POP".

$$
\begin{array}{l}
\quad \begin{array}{cccccccccccc} \text{3MA-2} & & & \text{3MB-2} & & & \text{3MC-2} & & & \text{3MD-2} & & \end{array} \\
\begin{array}{l} \text{3MA-2} \\ \\ \\ \text{3MB-2} \\ \\ \\ \text{3MC-2} \\ \\ \\ \text{3MD-2} \\ \\ \\ \end{array}
\left[\begin{array}{ccc:ccc:ccc:ccc}
+ & + & + & + & + & + & + & + & + & + & + & + \\
+ & + & + & + & + & + & + & + & + & + & + & + \\
+ & + & + & + & + & + & + & + & + & + & + & + \\
\hdashline
+ & + & + & + & + & + & + & + & + & + & + & + \\
+ & + & + & + & + & + & + & + & + & + & + & + \\
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\hdashline
+ & + & + & + & + & + & + & + & + & + & + & + \\
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+ & + & + & + & + & + & + & + & + & + & + & + \\
\hdashline
+ & + & + & + & + & + & + & + & + & + & + & + \\
+ & + & + & + & + & + & + & + & + & + & + & + \\
+ & + & + & + & + & + & + & + & + & + & + & +
\end{array}\right]
\end{array}
$$

Fig. 6 Partitioned Element Stiffness Matrix and Addresses in the Total Stiffness Matrix

The distribution of the nonzero elements is dependent upon the way the nodes of the finite element model are numbered. Because of the symmetry of the stiffness matrix, only the lower or upper triangular matrix is considered. For the purpose of this discussion definitions of the following terms are in order. The gross population (P_{gross}) of the stiffness matrix is defined as the total number of elements in the upper triangle of the matrix. The net population (P_{net}) is the total number of non-zero elements in the upper triangle. Zeros resulting from transformations are not excluded from the net population. The apparent population ($P_{apparent}$) is the actual number of elements considered as nonzeros by a given solution scheme. From these definitions

$$P_{net} \leq P_{apparent} \leq P_{gross} \tag{59}$$

For a given structure P_{gross} and P_{net} are invariant and are given by

$$P_{gross} = \frac{N\ (N + 1)}{2} \tag{60}$$

and

$$P_{net} = \frac{n\ (n + 1)}{2}\ (\text{number of nodes}) + \sum_{i=1}^{m} \frac{n^2[k_i(k_i-1)]}{2} - n^2(NR) \tag{61}$$

where N is the total number of degrees of freedom of the structure, n is the number of degrees of freedom of each node (all the nodes are assumed to have the same number of degrees of freedom; when this is not true the necessary modification is simple), k_i is the number of nodes to which the i^{th} element is connected, and m is the number of elements in the structure. The quantity NR is given by

$$NR = \sum_{i=1}^{p} (b_i - 1) \tag{62}$$

where b_i is the number of elements connecting the same pair of nodes and p is the total number of pairs of directly connected nodes. If the structure consists of bar and/or beam elements only, NR is zero. For the example shown in Figure 6a, the value of NR is 3.

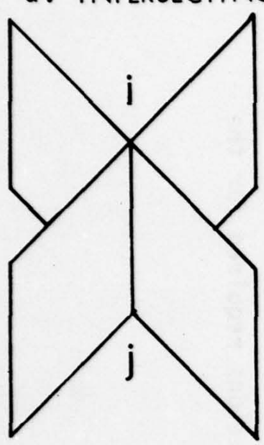
The quantity $P_{apparent}$ is dependent on the nature of the solution scheme used. For Gaussian elimination with no pivoting (LDL^T), $P_{apparent}$ may be defined as

$$P_{apparent} = \sum_{j=1}^{N} Q_j \tag{63}$$

where $Q_j = j - R_j + 1$ and where R_j is the row number of the first nonzero element in the j^{th} column. The solution scheme is most efficient when $P_{apparent} = P_{net}$. However, in large practical structures this condition is difficult to attain.

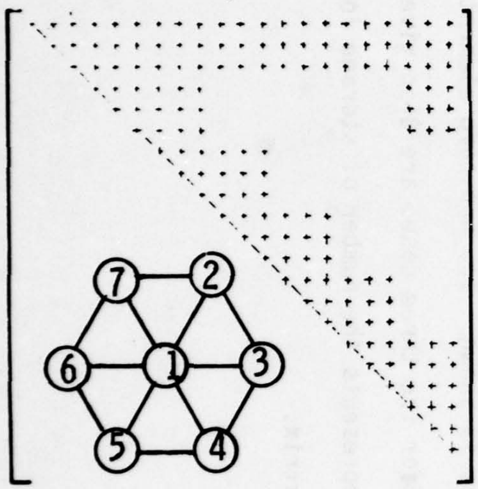
The value of $P_{apparent}$ changes with the node numbering scheme of the finite element model. The example shown in Figure 7 illustrates this point. A seven node three dimensional bar structure (n = 3) is numbered in three different ways and the resulting effect on the respective stiffness matrices is shown. The non-zero elements are marked by (+). The populations for the three cases are also given in the same figure. $P_{apparent}$ represents the number of storage locations required for the stiffness matrix.

a. INTERSECTING PLATES

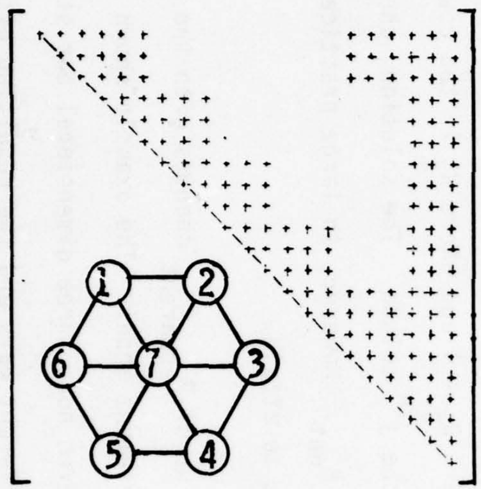


NR = 3

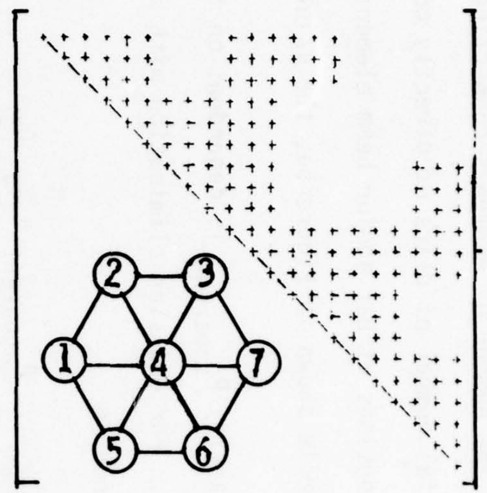
SCHEME NO.	P_{GROSS}	P_{NET}	$P_{APPARENT}$
1	231	150	231
2	231	150	177
3	231	150	177



SCHEME 1



SCHEME 2



SCHEME 3

FIGURE 7: DISTRIBUTION OF NONZERO ELEMENTS IN THE STIFFNESS MATRIX

Subroutine "ELSTIC"

This routine generates the 3 x 3 elastic constants matrix for a given material (see Eq. 3).

Subroutine "COORD"

This routine establishes the local coordinate system for all the elements and also determines the nodal coordinates in the local system. It generates the direction cosine matrix which will be used to transform the element stiffness matrices to the global coordinate system (see Eqs. 13 and 16).

i. Bar Element

The local coordinate system of the bar element is established by drawing a line between the two nodes MA and MB (see Fig. 2) connecting the bar. The direction cosines are determined by

$$X_{Comp} = X_{MA} - X_{MB}$$

$$Y_{Comp} = Y_{MA} - Y_{MB} \tag{64}$$

$$Z_{Comp} = Z_{MA} - Z_{MB}$$

$$L = (X^2_{Comp} + Y^2_{Comp} + Z^2_{Comp})^{1/2} \tag{65}$$

$$\ell_1 = \frac{X_{Comp}}{L} \quad m_1 = \frac{Y_{Comp}}{L} \quad n_1 = \frac{Z_{Comp}}{L} \tag{66}$$

where X_{MA}, Y_{MA} and Z_{MA} are the three coordinates of the node MA in global coordinate system. The direction cosines ℓ_1, m_1, and n_1 become the first row of the 3 x 3 matrix $\underset{\sim}{A}$.

ii. Triangular Membrane Element

The local coordinate system of the triangular membrane element is established by assigning the local x-axis to the line joining nodes MA and MB. The direction cosines of this line are determined as in the case of the bar element. The plane of the plate is established by two unit vectors in the directions of the lines joining nodes MA-MB and MA-MC. If $\hat{a}$ and $\hat{b}$ are these two unit vectors, then the normal to the plane is obtained by

$$\hat{a} \times \hat{b} = \vec{c} \tag{67}$$

Since $\hat{a}$ and $\hat{b}$ are not orthogonal vectors, $\vec{c}$ is not a unit vector. The unit vector in this direction is given by

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} \tag{68}$$

The local z-axis is in the direction of the unit vector $\hat{c}$. Now the local y-axis is established by

$$\hat{c} \times \hat{a} = \hat{d} \tag{69}$$

The direction cosines of x and y become the first two rows of matrix $\underset{\sim}{A}$.

iii. Quadrilateral Membrane and Shear Panel

The local coordinate system of the quadrilateral membrane and the shear panel are established by a procedure similar to that of the triangle. The plane of the triangle connecting the three nodes MA, MB, and MC becomes the reference plane. Any warping in the quadrilaterals and shear panels is ignored. If there is too much warping in the quadrilaterals, it is better to divide them into two or more triangles or reduce the mesh size. In the case of excessively warped shear panels, the size of the grid must be

reduced. "ANALYZE" does not have a provision for determining the warp and the consequent kick forces.

The node MA of the element becomes the origin of the element local coordinate system and the coordinates of the remaining nodes are determined by expressions similar to the following:

$$x_3 = (X_{MC} - X_{MA})\ell_1 + (Y_{MC} - Y_{MA})m_1 + (Z_{MC} - Z_{MA})n_1$$

$$y_3 = (X_{MC} - X_{MA})\ell_2 + (Y_{MC} - Y_{MA})m_2 + (Z_{MC} - Z_{MA})n_2$$

This subroutine also determines the coordinates of the fictitious node needed to break the quadrilateral and shear panels into four triangles. This interior node is established by

$$x_5 = \frac{x_1 + x_2 + x_3 + x_4}{4}$$
$$y_5 = \frac{y_1 + y_2 + y_3 + y_4}{4} \tag{70}$$

where x_1, x_2 ....x_5 and y_1, y_2 ....y_5 are the coordinates of the five nodes (including the fictitious interior node) of the quadrilaterals and shear panels in the local coordinate system.

Subroutine "ELSTIF"

This subroutine determines the stiffness matrix of the bar by Eq. 22. It also transforms the bar stiffness matrix to the global coordinate system by

$$\underset{\sim}{K}_i = \underset{\sim}{a}_i^t \ \underset{\sim}{k}_i \ \underset{\sim}{a}_i \tag{71}$$

Subroutines "PLSTIF" and "CRAMER"

The routine "PLSTIF" determines the element stiffness matrix of the triangle in the local coordinate system. This is also the basic routine for determining the stiffness matrices of the four triangles of the quadrilateral and the shear panel.

"PLSTIF" first calls the routine "CRAMER", which determines the inverse of the matrix $\underset{\sim}{X}$ by Cramer's rule. The matrix $\underset{\sim}{X}$ is given by Eq. 34. The determinant of $\underset{\sim}{X}$ represents twice the area of the triangle.

Then the "PLSTIF" subroutine determines the element stiffness matrix by Eq. 40. In determining the matrices $\underset{\sim}{\varepsilon}^{(i)}$ and $\underset{\sim}{\varepsilon}^{(j)}$, it takes advantage of the fact that the columns of Z^{-1} (see Eq. 33) represent unit displacement modes (see explanation under Eq. 34).

In computing the stiffness matrices of the triangles of the shear panels, "PLSTIF" considers only the shear strain energy. For example, in such a case, Eq. 40 becomes

$$\underset{\sim}{k} = \frac{1}{2}\left|\underset{\sim}{X}\right| t \begin{bmatrix} \varepsilon_{xy}^{(1)} G \varepsilon_{xy}^{(1)} & \varepsilon_{xy}^{(1)} G \varepsilon_{xy}^{(2)} & \cdots & \varepsilon_{xy}^{(1)} G \varepsilon_{xy}^{(6)} \\ \vdots & & & \vdots \\ \varepsilon_{xy}^{(6)} G \varepsilon_{xy}^{(1)} & \varepsilon_{xy}^{(6)} G \varepsilon_{xy}^{(2)} & \cdots & \varepsilon_{xy}^{(6)} G \varepsilon_{xy}^{(6)} \end{bmatrix} \qquad (72)$$

Subroutine "QDRLTL"

This subroutine simply manages the routines "PLSTIF", "SUM", and "CONDNS" in computing the stiffness matrix of the quadrilateral membrane and shear panel. This routine also makes provision for assigning different sides as reference axis for the shear panels.

Subroutine "SUM"

This subroutine adds the four triangle stiffness matrices computed by "PLSTIF" to produce a 10 x 10 stiffness matrix (including two degrees of freedom for the interior node) for the quadrilateral or shear panel.

Subroutine "CONDNS"

This routine condenses the 10 x 10 quadrilateral or shear panel stiffness matrix to an 8 x 8 matrix. The condensation is done by using Eq. 56.

Subroutine "CHANGE"

This routine interchanges the rows and columns of the quadrilateral (or shear panel) stiffness matrix so that the element degrees of freedom are in ascending order before addition to the structure stiffness matrix. This step is necessary because the routine "ASEMBL" assumes that the element degrees of freedom are in ascending order.

Subroutine "TRNSFM"

This routine transforms the plate element stiffness matrices from the local to the global coordinate system by (see Eq. 16)

$$\underset{\sim}{K}_i = \underset{\sim}{a}_i^t \; \underset{\sim}{k}_i \; \underset{\sim}{a}_i \tag{73}$$

where $\underset{\sim}{K}_i$ is the transformed element stiffness matrix of the i^{th} element.

Subroutine "ASEMBL"

This routine adds the element stiffness matrices to the total stiffness matrix.

$$\underset{\sim}{K} = \sum_{i=1}^{m} \underset{\sim}{K}_i \tag{74}$$

For an explanation of the rules of this addition see the description of subroutine "POP". It should be noted that only the upper half of the stiffness matrix is stored. This storage is columnwise starting with the first non-zero element above the diagonal.

Subroutine "PRINTK"

The purpose of this routine is to print the stiffness matrix (if desired) rowwise starting with the first non-zero element and proceeding to the diagonal.

Subroutine "BOUND2"

This routine eliminates the rows and columns corresponding to the constrained degrees of freedom and condenses the stiffness matrix.

Subroutine "REDUCE"

This routine eliminates the rows of the applied force matrix corresponding to the constrained degrees of freedom. It is assumed that each column of the force matrix represents an independent load condition.

Subroutine "GAUSS"

"GAUSS" solves the load deflection equations (Eq. 17) by Gaussian elimination. The first step of the solution is the decomposition of the stiffness matrix by Eq. 18. The next two steps represent forward and back substitution using Eqs. 19 and 20 respectively. For the solution of additional load vectors only the steps FBS have to be repeated. If "GAUSS" is entered with any value other than 0 for the parameter NDCOMP, only the last two steps will be executed. The matrices $\underset{\sim}{L}$ and $\underset{\sim}{D}$ are stored in place of the original stiffness matrix.

Subroutine "RESTOR"

This routine restores the displacement or force matrix to full size by assigning zero values to boundary degrees of freedom.

Subroutine "ELFORC"

This routine extracts the element displacements from the global coordinate system and transforms them to the local coordinate system by Eq. 13.

Subroutine "STRESS"

The purpose of the "STRESS" routine is to compute strains and stresses in the triangular element. It first calls the routine "CRAMER" which computes $\underset{\sim}{X}^{-1}$ (Eq. 34) by Cramer's rule. The strains in the element are then calculated by Eqs. 30 and 35 thru 37. The stresses in the element are computed by Eq. 2. Also it computes the strain energy and the effective stress in the element by Eqs. 1 and 45 respectively.

Subroutine "QLSTRS"

This routine prepares the data for computing stresses in the four triangles of the quadrilateral or shear panel elements. First it determines the interior node displacements from the corner node displacements using Eq. 54. Then it calls subroutine "STRESS" to compute the stresses in the four triangles. It adds the strain energy of the four triangles to obtain the total strain energy. It identifies the triangle with the largest effective stress and normalizes the effective stress of the three remaining triangles with respect to this largest value.

Subroutine "PRNTDR"

This subroutine prints out the table of node information. This includes the node number, its coordinates, applied forces, and the displacements.

NAME	NUMBER OF CARDS	CALLED FROM
ANALYZE	315	Main Program
POP	62	ANALYZE
ELSTIC	15	ANALYZE
COORD	44	ANALYZE
ELSTIF	21	ANALYZE
PLSTIF	46	ANALYZE, QDRLTL
CRAMER	19	PLSTIF, STRESS
QDRLTL	32	ANALYZE
SUM	23	QDRLTL, QLSTRS
CONDNS	36	QDRLTL, QLSTRS
CHANGE	25	CONDNS
TRNSFM	36	ANALYZE
ASEMBL	41	ANALYZE
PRINTK	15	ANALYZE
BOUND2	35	ANALYZE
REDUCE	18	ANALYZE
GAUSS	57	ANALYZE
RESTOR	28	ANALYZE
ELFORC	22	ANALYZE
STRESS	33	ANALYZE, QLSTRS
QLSTRS	65	ANALYZE
PRNTDR	39	ANALYZE
TOTAL	1027	

Table 1: Program Description

6. INPUT INSTRUCTIONS

Input for the programs is divided into a number of card sets. Each card set will consist of one or more cards. Only three Formats are used for input. An integer Format (14I5), a floating point Format (6F10.0) and a mixed Format 3(F10.0,2I5). The first four card sets will each have one card regardless of the size of the problem. The number of cards required for the remaining card sets depends on the problem size. The first card set indicates the number of problems (structures) to be analyzed. If this number is more than one, the program assumes that the remaining card sets will be supplied for each problem one after the other. The next card set is for the title of the problem. Card set three defines the basic parameters like the number of elements, nodes etc. And set 4 defines the properties of a reference material. This material can be any one of the materials used. The remaining card sets define material properties (5 and 6), type of elements (7), element connections (8, 9, 10, 11), sizes of the elements (12), element-material identification (13), node coordinates (14), boundaries (15) and loading information (16 and 17).

INPUT INSTRUCTION DETAILS

CARD SET (FORMAT)	PARAMETER	DESCRIPTION
1 (14I5)	NSTR	Number of data sets
2 (8A10)	TITLE	An alphanumeric description of the problem to be solved.
3 (14I5)	MEMBS	Number of elements
	JOINTS	Number of nodes
	NBNDRY	Number of restrained degrees of freedom
	LOADS	Number of loading conditions
	MM	MM =2 Two dimensional problem; =3 Three dimensional problem
	NR	Variable used only for calculating the net population of the stiffness matrix. It has no other role in the program. See Section 5.
	INCHES	INCHES =1 Coordinate data is in inches; ≠1 Coordinate data is in feet
	KIPS	KIPS =1 Applied forces are in kips; ≠1 Applied forces are in pounds
	NMAT	Number of materials
	MSSTRS	MSSTRS =0 Margin of safety calculated from default allowable stresses.; ≠0 Margin of safety calculated from input allowable stresses.
4 (6F10.5)	EEE	YOUNG'S modulus/10^6 of one of the elements in psi.
	PMU	POISSON'S ratio of one of the elements.
	RHO	Density of one of the elements in lbs/in^3.

CARD SET (FORMAT)	PARAMETER	DESCRIPTION
	IF MSSTRS = 0, skip CARD SET 5.	
5 (6F10.5)	ALSTRS(I) I=1,...,3*NMAT	Allowable stresses/10^3 in tension, compression and shear for the I^{th} material.
	IF NMAT ≠ 1, CARD SET 4 parameters can be for any of the materials. IF NMAT = 1, skip CARD SET 6.	
6 (6F10.5)	YOUNGM(I)	YOUNG'S modulus/10^6 for the I^{th} material in psi.
	POISON(I)	POISSON'S ratio for the I^{th} material.
	RHO1(I) I=1,...,NMAT	Density for the I^{th} material in lbs/in^3.
7 (14I5)	NNODES(I),I=1,...,MEMBS	Element Type NNODE(I) =2 BAR; =3 TRIANGLE; =4 QUADRILATERAL MEMBRANE; =5 SHEAR PANEL
8 (14I5)	MA(I),I=1,...,MEMBS	First node number of each element.
9 (14I5)	MB(I),I=1,...,MEMBS	Second node number of each element.
10 (14I5)	MC(I),I=1,...,MEMBS	Third node number of each element.
11 (14I5)	MD(I),I=1,...,MEMBS	Fourth node number of each element.

NOTE: For bars leave MC(I) and MD(I) blank. For triangles leave MD(I) blank. For each element let MA(I) be the lowest node number and MB(I) be the next lowest. For Quadrilaterals and Shear Panels, MC(I) and MD(I) are determined by continuing in the direction defined by MA(I) and MB(I).

CARD SET (FORMAT)	PARAMETER	DESCRIPTION
12 (6F10.5)	TH(I),I=1,...,MEMBS	Thickness of each element. For a bar thickness is cross-sectional area.
	IF NMAT =1, skip CARD SET 13.	
13 (14I5)	MYOUNG(I), I=1,...,MEMBS	Material number of each element.

CARD SET (FORMAT)	PARAMETER	DESCRIPTION
14 (6F10.5)	X(I)	X coordinate of the I^{th} node.
	Y(I)	Y coordinate of the I^{th} node.
	Z(I) I=1,...,JOINTS	Z coordinate of the I^{th} node.
	IF MM=2, only X(I) and Y(I) are input.	
15 (14I5)	IBND(I),I=1,...,NBNDRY	Degree of freedom numbers of those nodes which are restrained. For node K the degree of freedom numbers are 3*K-2, 3*K-1, and 3*K for MM=3 and 2*K-1, 2*K for MM=2.
16 (14I5)	NJLODS(I),I=1,...,LOADS	Number of load components in the I^{th} loading condition.
17 3(F10.0,2I5)	TFR(J)	Value of the load.
	IM(J)	Direction of the load IM(J) =1 x direction =2 y direction =3 z direction
	JM(J) J=1,...,NJLODS(I)	Number of the node where the load is applied.

7. OUTPUT DESCRIPTION

The primary output of the program ANALYZE consists of two tables (items 6 and 8 of the output description details). The first table gives element information and the second table gives information about the nodes. The element information includes member number, thickness (cross-sectional area of the bars), planform area (length of a bar), element type, stress information, strain energy, and margin of safety. The information about the nodes includes node (joint) number, node coordinates, applied forces, and the resulting displacements. In addition to these two tables output 3a (coming from subroutine POP) gives important information about the population distribution of the stiffness matrix. The value of the apparent population is crucial in determining the dimension of the stiffness matrix (SK). This dimension must be at least as big as or bigger than this value.

Item 7 gives information about the total strain energy (U) and the work of the external forces (W) for the structure. This information can be very useful for an equilibrium check.

Item 4 gives the weight of the structure. The remaining information is not really very important to the user.

OUTPUT DESCRIPTION DETAILS

Output for Program ANALYZE consists of the following:

1) Untitled echo of all input data except boundaries and applied loads.

2) Boundary data, i.e. contents of array IBND (CARD SET 15)

3) Output from Subroutine POP concerning the distribution of elements in the stiffness matrix. This information is generated before the stiffness matrix of the structure is assembled.

(a) Gross Population - total number of elements in the upper triangle of the matrix.

Net Population = actual population of possible non-zero elements in the upper triangle of the stiffness matrix. This number would be correct only if NR is correct in CARD SET 3.

Apparent Population = actual number of elements considered as non-zero by a given solution scheme. Thus the apparent population represents the number of storage locations required for the stiffness matrix.

(b) Starting Row Numbers for each column - the number of the row where the first non-zero element occurs in each column.

(c) Number of Diagonal Elements in Single Array Stiffness Matrix. For each Column I the actual number of elements, ID(I), in the upper triangular matrix up to and including that column, i.e.

$$ID(I) = \frac{I(I+3)}{2} - \sum_{j=1}^{I} b_j$$

where b_j is the row number given for Column I in (b). Thus for the last column, ILAST,

ID(ILAST) = Apparent Population

4) Weight of the structure

5) Boundary conditions applied to the stiffness matrix alter the arrays defined by (b) and (c) above, and thus they are reprinted.

6) Output for each element after analysis.

(a) MEMB - Element Number

(b) THICK - Thickness of the element. For a bar thickness is cross-sectional area.

(c) AREA - Area of the element. For a bar area is length.

(d) TYPE - Type is a composite number which describes the element type and material number. Type is defined as

TYPE = NNODES(I)x10 + MYOUNG(I).

See CARD SETS 7 and 13.

Note: If the number of materials is greater than 10, TYPE is meaningless. If the number of materials is 1, MYOUNG(I)=1 for all I.

(e) MA, MB, MC, MD - defined in CARD SETS 8,9,10, and 11.

(f) SIGMA-X (σ_x), SIGMA-Y (σ_y), SIGMA-XY (σ_{xy}).

Stresses in the x-y local coordinates of the element.

EFSTR-1, EFSTR-2, EFSTR-3, EFSTR-4 - Effective stresses in the element determined by the Von Mises Criterion.

The stress output varies per element type.

(i) BAR SIGMA-X only

(ii) TRIANGLE SIGMA-X, SIGMA-Y, SIGMA-XY, EFSTR-1

(iii) QUADRILATERAL MEMBRANE

The Quadrilateral membrane element is divided into 4 triangles for analysis. SIGMA-X, SIGMA-Y, SIGMA-XY are for that triangle with the maximum effective stress. This maximum effective stress is given as EFSTR-i for some i, i=1,...,4. Then EFSTR-j, j≠i, are defined as the ratio of the effective stress for triangle j to the maximum effective stress.

(iv) SHEAR PANEL

The Shear Panel is also divided into 4 triangles for analysis. SIGMA-XY (τ_{xy}) is for that triangle with the maximum effective stress. Then EFSTR-i, i=1,...,4 are as defined in (iii).

(g) ENERGY - Total strain energy in the element.

(h) MS - Margin of Safety for the element.

NOTE: If the number of loading conditions is greater than 1, output (f) and (h) are given continuously for each load case.

7) The total strain energy (U) of the structure and the work (W) of the external forces for each loading condition.

8) Output for each node after analysis.

   (a) JOINT - Node Number

   (b) X, Y, Z - x, y, and z coordinate of the node

   (c) FORCE-X, FORCE-Y, FORCE-Z - applied forces in the x, y, and z direction.

   (d) DISPL-X, DISPL-Y, DISPL-Z - Displacements in the x, y, and z direction.

NOTE: If the number of loading conditions is greater than 1, output (c) and (d) are given continuously for each load case.

8. SAMPLE PROBLEM

A three spar wing shown in Figure 8 is idealized by membrane quadrilaterals, shear panels, and bars (axial force members). The top and bottom skins are represented by membrane quadrilaterals and the spars and ribs by shear panels. In addition, the top and bottom nodes are connected by bar elements or posts. The root section of the wing is assumed to be fixed. The finite element model of the wing box is shown in Figure 8.

The wing is analyzed for two independent loads. These loading conditions are generated by simplified pressure distributions representative of a subsonic, forward-center-of-pressure loading, and a supersonic near-uniform-pressure loading. The material properties (aluminum) are given in Figure 9.

The results of the analysis are given in Appendix D. They are given in two tables. The first table gives element information. This includes sizes, connections, stresses, energies, and margins of safety. The second table gives information about nodes. This includes coordinates, applied loads, and displacements. For a detailed description of the complete output, see Section 7.

The wing was also analyzed by the NASTRAN program. Table 2 compares ANALYZE and NASTRAN z-displacements.

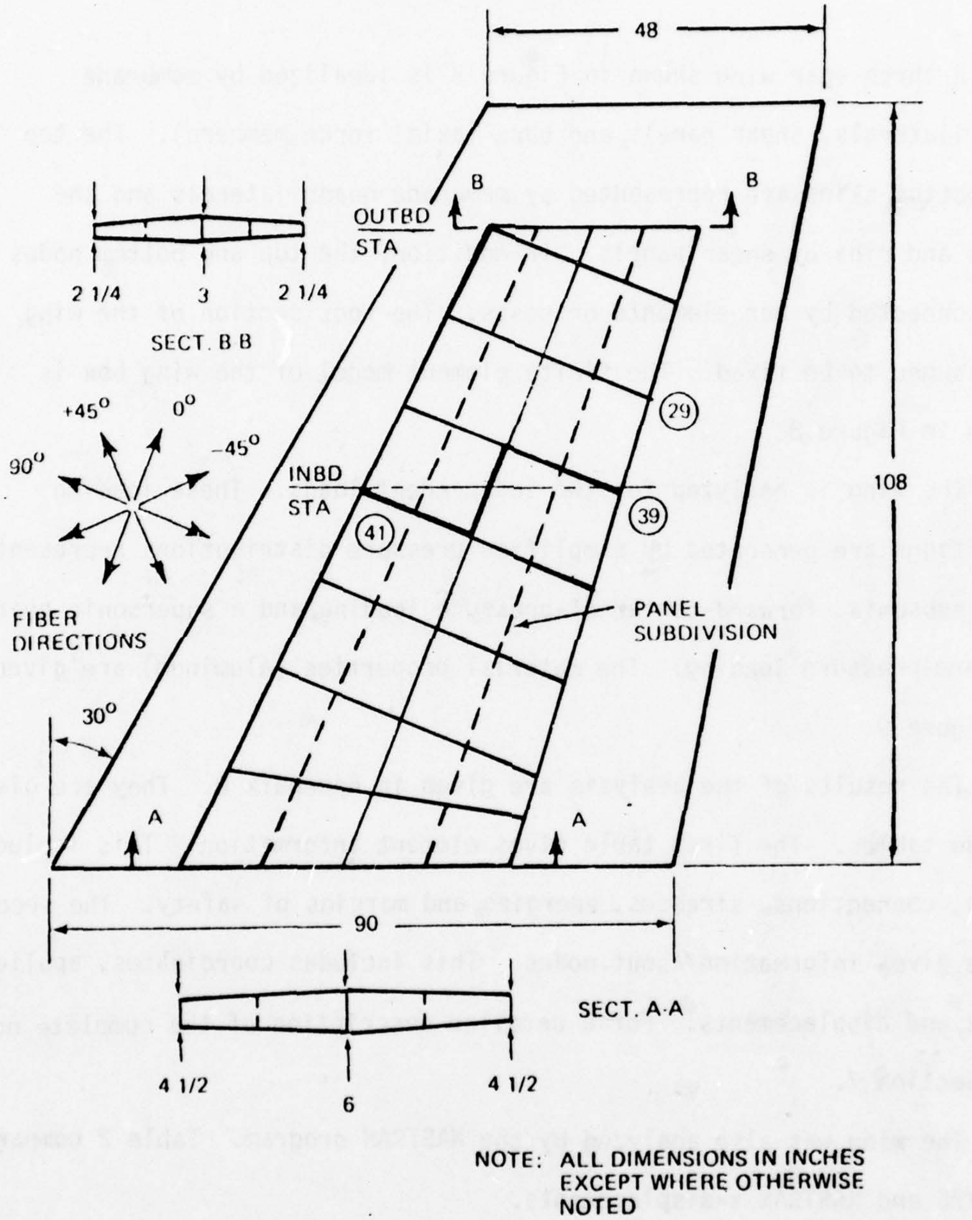


Figure 8. Aerodynamic Planform and Primary Structural Arrangement of Wing

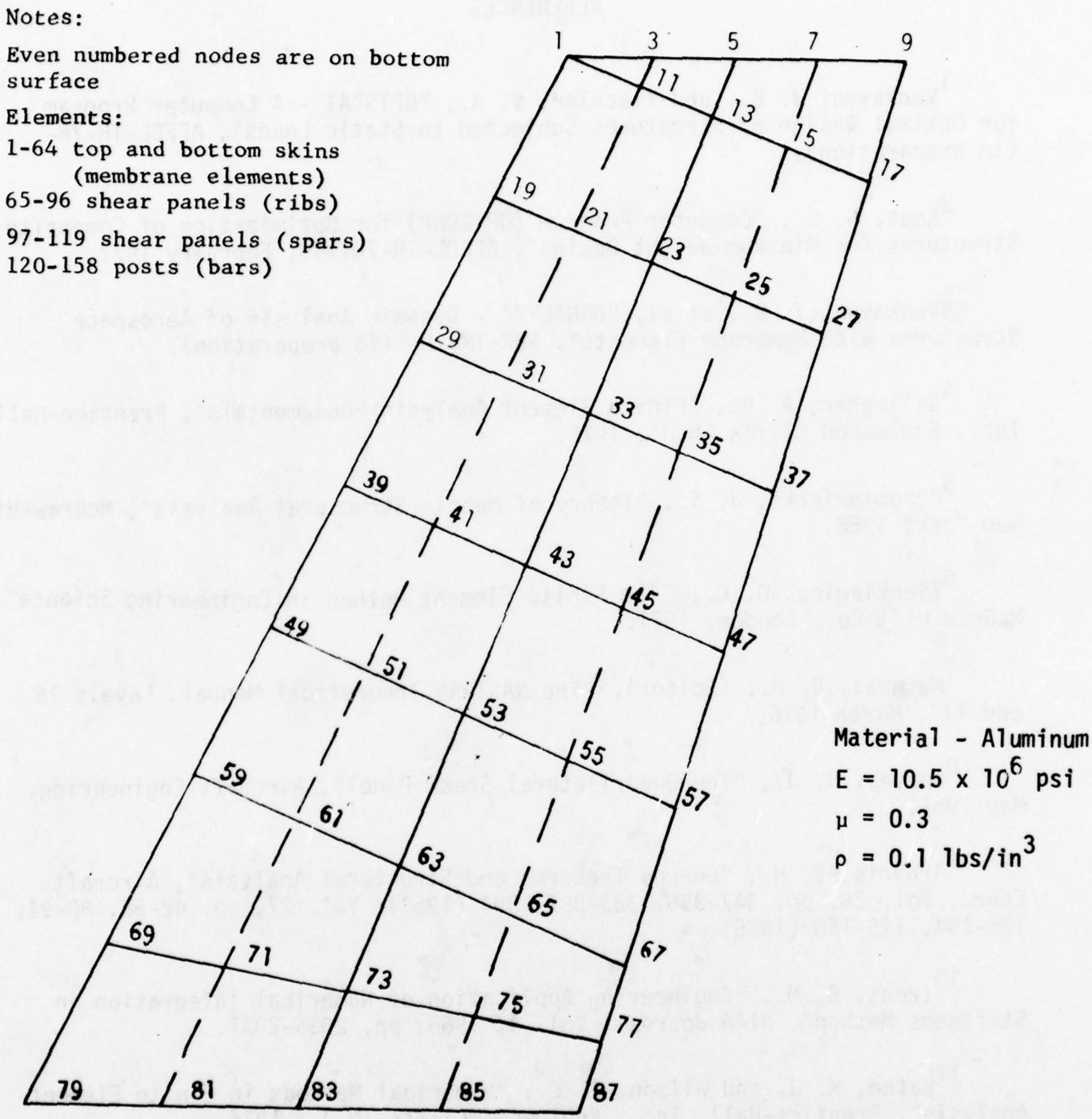


Figure 9. Finite Element Representation of Wing Box

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APPENDIX A: ESTIMATION OF CORE REQUIREMENTS

The purpose of this appendix is to aid in the approximate estimation of core requirements for the program. These change with the problem size. For example with $100k_8$ (120K bytes or 30K decimal) it is possible to solve problems of the size 250 to 300 degrees of freedom assuming that the nodes are numbered with reasonable care for an optimum stiffness profile (See the discussion under subroutine "POP" in Section 5). The dimensional requirements of various arrays are explained by comment cards at the beginning of the program. However, this section reiterates the importance of adjusting the dimensions of some important arrays.

The arrays can be grouped into nine types. The number of elements, degrees of freedom, loading conditions, the number of boundaries and the number of materials are some of the variables that affect the size of the arrays. The arrays must be dimensioned at least as big or bigger than the number of these variables in the problem. The arrays with fixed sizes (not affected by problem size) are dimensioned first. The total core requirement of these arrays is relatively small. Next, the arrays that depend on the number of materials are dimensioned. The third group consists of a single array IBND which is dimensioned according to the number of boundary conditions. The fourth group varies with the number of loading conditions. In this case the dimension of the single arrays is equal to the number of loading conditions. For rectangular arrays, the first dimension is fixed, and the second dimension represents the number of loading conditions. The fifth group varies with the number of elements. The sixth group depends on the number of nodes.

The number of degrees of freedom determines the dimensions of the seventh group. The number of degrees of freedom and the loading conditions determine the size of the eighth group of arrays. The first dimension of these arrays represents the degrees of freedom, and the second dimension represents the loading conditions. The SK matrix in the last group depends on the number of degrees of freedom and the profile of the total structure stiffness matrix which in turn depends on the ordering of the node numbers (See the discussion under subroutine "POP" in Section 5).

The preliminary estimate of the size of the SK array can be based on the estimation of the semi-bandwidth. This would be an upper bound for the dimension of SK. The actual dimension of SK can be determined after passing through the subroutine "POP". This routine gives a number for the apparent population of the stiffness matrix from the information of the element connections. SK must be dimensioned at least as big or bigger then the apparent population in order to solve the problem. Usually SK is the largest array in the program, and its size can be reduced by numbering the nodes for the optimum profile of the stiffness matrix. In absence of an adequate procedure for optimization of this profile, some sort of bandwidth optimization is acceptable. It should be noted that the value of the variable MAXSK (defined in the beginning of the program) should be the same as the dimension of SK. When the dimension of SK is changed, the value of MAXSK should also be changed.

The next largest arrays are FR and DR. They represent the applied force and the computed displacement matrix respectively. The dimension

of the arrays depends on the number of degrees of freedom and the independent loading conditions. The first dimension should be at least as big or bigger than the number of degrees of freedom of the problem. Similarly the second dimension is determined by the number of loading conditions. In addition the first dimension should be the same as the variable NNMAX defined in the beginning of the program. Whenever the dimensions of FR and DR are changed, NNMAX must also be changed accordingly.

The arrays ICOL and IDIAG depend on the number of degrees of freedom of the problem. Together they identify the profile of the stiffness matrix. For instance, ICOL(I) gives the row number of the first non-zero element in the I^{th} column of the stiffness matrix. IDIAG(I) gives the address of the diagonal element of the I^{th} column of the stiffness matrix in the single array SK.

The arrays MA, MB, MC and MD are assigned for element connections. NNODES is for the type of elements. The array TH is for the sizes (thickness of plate elements and cross-sectional area of bars) of the elements. The array MYOUNG identifies the material type of the elements. The remaining arrays are small and have minor influence on the core requirements.

Frequent Errors Encountered in Using "ANALYZE"

1. The element connections MA, MB, MC and MD must be specified by starting with the lowest node number for MA and the next lowest, but adjacent node number, for MB. MC and MD are then defined by continuing in the direction established by MA and MB. See the description of card sets 8, 9, 10, and 11 in the input instructions, Section 6.

2. The boundary degrees of freedom (IBND) must be in ascending order. See the description of card set 15 in Section 6, Input Instructions.

3. The first dimension of FR and DR must be the same as the value of the variable NNMAX (defined at the beginning of the program).

4. The value of MAXSK (defined at the beginning of the program) must be equal to or greater than the value of the apparent population given by the routine "POP". The dimension of the array SK must be equal to the value given for MAXSK.

5. The sides of the shear panels must be attached to one or more normal stress carrying elements such as posts (bars), membrane quadrilaterals or triangles.

APPENDIX B : LISTING OF THE PROGRAM


```
      PROGRAM ANALYZE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)              ANALYZE
C                                                                         ANALYZE
C     THE FOLLOWING DIM ARE FOR INTERNAL USE                              ANALYZE
      DIMENSION AA(3,3),EK(12,12),B(12,12),C(12,12),XI(5),ETA(5),EE(3,3)  ANALYZE
     1          ,MAA(4),MBB(4),MCC(4),EKK(12,12),TRANG(4),TFR(10),        ANALYZE
     2          IM(10),JM(10),ALS(3),TITLE(8)                             ANALYZE
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF MATERIALS                ANALYZE
      DIMENSION YOUNGM(20),POISON(20),RHO1(20)                            ANALYZE
C     THE FOLLOWING DIM ARE THREE TIMES THE NUMBER OF MATERIALS           ANALYZE
      DIMENSION ALSTRS(60)                                                ANALYZE
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF BOUND COND (NBNDRY)      ANALYZE
      DIMENSION IBND(50)                                                  ANALYZE
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF LOADING CONDITIONS (L)   ANALYZE
      DIMENSION EDDR(12,5),SSX(4,5),SSY(4,5),SSXY(4,5),SXY(5),KTR(5),     ANALYZE
     1          EFSTRS(5),EFFSTR(4,5),EDR(12,5),SX(5),SY(5),NJLOBS(5),    ANALYZE
     2          ELEENG(5),ENGTOT(5),ENGSTR(5),ESR(5),SFTM(5)              ANALYZE
C     IF THE NUMBER OF LOADING CONDITIONS EXCEED 10, THEN CHANGE THE      ANALYZE
C     DIMENSION OF EX, EY, EXY IN SUBROUTINE STRESS, ENGG IN SUBROUTINE   ANALYZE
C     OLSTRS AND TDR1, TDR2 IN SUBROUTINE RESTOR                          ANALYZE
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF ELEMENTS                 ANALYZE
      DIMENSION MA(160),MB(160),MC(160),MD(160),TH(160),NNODES(160),      ANALYZE
     1          MYOUNG(160)                                               ANALYZE
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF JOINTS                   ANALYZE
      DIMENSION X(90),Y(90),Z(90)                                         ANALYZE
C     THE FOLLING DIM PERTAIN TO THE NUMBER OF DEG OF FREEDOM (NN)        ANALYZE
      DIMENSION IDIAG(270),ICOL(270)                                      ANALYZE
C     THE FOLLOWING DIM PERTAIN TO THE NUMBER OF DEG OF FREEDOM (NN)      ANALYZE
C     AND THE NUMBER OF LOADING CONDITIONS (L)                            ANALYZE
      DIMENSION DR(270,5),FR(270,5)                                       ANALYZE
C     THE FOLLOWING DIM PERTAINS TO THE TOTAL STIFFNESS MATRIX (SK)       ANALYZE
      DIMENSION SK(9110)                                                  ANALYZE
C                                                                         ANALYZE
C*************************************************************            ANALYZE
C                                                                         ANALYZE
C     THIS PROGRAM WAS DEVELOPED                                          ANALYZE
C                                                                         ANALYZE
C               DR. VIPPERLA B. VENKAYYA                                  ANALYZE
C               AIR FORCE FLIGHT DYNAMICS LABORATORY (AFFDL/FBR)          ANALYZE
C               WRIGHT-PATTERSON AIR FORCE BASE,DAYTON,OHIO               ANALYZE
C                                                                         ANALYZE
C*************************************************************            ANALYZE
C                                                                         ANALYZE
      INTEGER TYPE                                                        ANALYZE
C     NNMAX MUST BE THE DIMENSION OF FR,DR,IDIAG,ICOL                     ANALYZE
      NNMAX = 270                                                         ANALYZE
C     MAXSK MUST BE EQUAL OR GREATER THAN THE DIM OF SK                   ANALYZE
      MAXSK = 9110                                                        ANALYZE
      READ(5,2)    NSTR                                                   ANALYZE
    1 READ(5,76)  (TITLE(I), I = 1,8)                                     ANALYZE
   76 FORMAT(8A10)                                                        ANALYZE
      WRITE(6,77)  (TITLE(I), I = 1,8)                                    ANALYZE
   77 FORMAT(5X,8A10)                                                     ANALYZE
      KSTR=1                                                              ANALYZE
      READ(5,2) MEMBS,JOINTS,NBNDRY,LOADS,MM,NR,INCHES ,KIPS,NMAT,MSSTRS  ANALYZE
      WRITE(6,2) MEMBS,JOINTS,NBNDRY,LOADS,MM,NR,INCHES,KIPS,NMAT,MSSTRS  ANALYZE
      READ(5,3)EEE,PMU,RHO                                                ANALYZE
      IF (RHO .LT. .00001) RHO=0.1                                        ANALYZE
```

```
      DO 7783 I = 1,NMAT
      KX = 3*(I-1) + 1
      ALSTRS(KX) = 60000.
      ALSTRS(KX+1) = 60000.
 7783 ALSTRS(KX+2) = 36000.
      IF (MSSTRS .EQ. 0) GO TO 7782
      KX = 3*NMAT
      READ(5,3) (ALSTRS(I), I = 1,KX)
      DO 7781 I = 1,KX
 7781 ALSTRS(I) = 1000.*ALSTRS(I)
 7782 CONTINUE
      ALS(1) = ALSTRS(1)
      ALS(2) = ALSTRS(2)
      ALS(3) = ALSTRS(3)
      WRITE(6,333) EEE,PMU,RHO,ALS(1),ALS(2),ALS(3)
  333 FORMAT(6F15.3)
      IF (NMAT .LE. 1) GO TO 7777
      READ(5,3) (YOUNGM(I),POISON(I),RHO1(I), I = 1,NMAT)
      DO 7784 I = 1,NMAT
      KX = 3*(I-1) + 1
      WRITE(6,333) YOUNGM(I),POISON(I),RHO1(I),ALSTRS(KX),
     1ALSTRS(KX+1),ALSTRS(KX+2)
 7784 CONTINUE
 7777 READ(5,2)(NNODES(I),I=1,MEMBS)
      READ(5,2)(MA(I),I=1,MEMBS)
      READ(5,2)(MB(I),I=1,MEMBS)
      READ(5,2)(MC(I),I=1,MEMBS)
      READ(5,2)(MD(I),I=1,MEMBS)
      READ(5,3)(TH(I),I=1,MEMBS)
      IF (NMAT .LE. 1) GO TO 7778
      READ(5,2) (MYOUNG(I), I =  1,MEMBS)
 7778 DO 5464 I=1,MEMBS
      IF (NMAT .EQ. 1) MYOUNG(I) = 1
      WRITE(6,33)I,NNODES(I),MYOUNG(I),MA(I),MB(I),MC(I),MD(I),TH(I)
 5464 CONTINUE
   33 FORMAT(7I5,4F10.5)
    2 FORMAT(14I5)
      EEE = EEE*(10.0**6)
      E = EEE
      E1=1.0
      IF (MM .LT. 3) GO TO 4
      READ(5,3)(X(I),Y(I),Z(I),I=1,JOINTS)
      GO TO 6
    4 READ (5,3) (X(I),Y(I),I=1,JOINTS)
      DO 11 I=1,JOINTS
   11 Z(I)=0.0
    3 FORMAT (6F10.5)
    6 CONTINUE
      IF(INCHES .EQ. 1)GO TO 9
 7000 FORMAT (20X, 3F10.4)
      DO 7 I=1,JOINTS
      X(I)=X(I)*12.0
      Z(I)=Z(I)*12.0
    7 Y(I)=Y(I)*12.0
    9 CONTINUE
      WRITE(6,7000) (X(I),Y(I),Z(I), I = 1,JOINTS)
      NN=MM*JOINTS
```

```
      NM=NN-NBNDRY
      READ(5,2) (IBND(I),I=1,NBNDRY)
      WRITE(6,5)
    5 FORMAT(1H1,///2X,10HBOUNDARIES ///)
      WRITE(6,1009)(IBND(I),I=1,NBNDRY)
      DO 10 I=1,NN
      DO 10 J=1,LOADS
      DR(I,J)=0
   10 FR(I,J)=0
      READ(5, 2) (NJLODS(I),I=1,LOADS)
      DO 21 J=1,LOADS
      KH=NJLODS(J)
   12 IF(KH-3) 13,13,14
   13 KX=KH
      GO TO 15
   14 KX=3
   15 READ(5,16) (TFR(I),IM(I),JM(I),I=1,KX)
   16 FORMAT (3(F10.0,2I5))
      DO 19 I=1,KX
      KY=MM*JM(I)-MM+IM(I)
   19 FR(KY,J)=FR(KY,J)+TFR(I)
      KH=KH-KX
      IF(KH)21,21,12
   21 CONTINUE
      IF(KIPS .NE. 1)GO TO 666
      DO 17 I=1,NN
      DO 17 J=1,LOADS
   17 FR(I,J)=1000.0*FR(I,J)
  666 CONTINUE
      CALL POP(MEMBS,JOINTS,MM,MA,MB,MC,MD,NNODES,ICOL,IDIAG,NONZRO,NR)
      IF(NONZRO .GT. MAXSK)GO TO 1000
      DO 8 I=1,NONZRO
    8 SK(I)=0
      CALL ELSTIC(1.0,PMU,EE)
      DO 120 I=1,4
      MAA(I)=I
      MBB(I)=I+1
  120 MCC(I)=5
      MAA(4)=1
      MBB(4)=4
      WEIGHT = 0.0
      DO 400 L = 1,MEMBS
      IF (NMAT .LE. 1) GO TO 20
      KX = MYOUNG(L)
      E = YOUNGM(KX)*10**6
      PMU = POISON(KX)
      E1 = E/EEE
      CALL ELSTIC(E1,PMU,EE)
   20 CALL COORD(MA(L),MB(L),MC(L),MD(L),X,Y,Z,AA,XI,ETA,AL,NNODES(L),0)
      IF(NNODES(L) -3)102,100,124
  124 CALL ODRLTL(EK,EKK,TH(L),QUAD,MA(L),MB(L),MC(L),MD(L),MAA,MBB,MCC,
     1XI, ETA,NNODES(L),EE,TRANG,0)
      GO TO 101
  100 CONTINUE
      CALL PLSTIF(EK ,TH(L),TRIANG, 1,2,3                 ,XI,ETA,EE,0.,0)
      QUAD = TRIANG
  101 CALL TRNSFM(EK,AA,B,C,MM,NNODES(L),12)
```

```
      GO TO 103
  102 CALL ELSTIF(AA,B,C,TH(L),MM,AL,E1)
      QUAD = AL
  103 CALL ASEMBL(SK,C,MA(L),MB(L),MC(L),MD(L),MM,IDIAG,NNODES(L),12)
   30 FORMAT(/1X,6E15.5/)
      IF (NMAT .LE. 1) GO TO 405
      KX = MYOUNG(L)
      RHO = RHO1(KX)
      IF(RHO1(KX) .LE. .00001) RHO=0.1
  405 WEIGHT = WEIGHT + TH(L)*QUAD*RHO
  400 CONTINUE
      WRITE(6,410) WEIGHT
  410 FORMAT(1H0,10X,25HWEIGHT OF THE STRUCTURE =,E15.5)
   35 CONTINUE
C     CALL PRINTK(SK,IDIAG,NN)
      CALL BOUND2(SK,IBND,NN,NBNDRY,IDIAG,ICOL)
      WRITE(6,1009)(ICOL(I),I=1,NM)
      WRITE(6,1009)(IDIAG(I),I=1,NM)
      NONZRO=IDIAG(NM)
 1009 FORMAT(1X,10I13)
      NDCOMP=0
      CALL REDUCE(FR,IBND,NN,NBNDRY,LOADS,NNMAX)
      CALL GAUSS(SK,FR,DR,ICOL,IDIAG,LOADS,NM,NNMAX,NDCOMP)
      IF(NDCOMP.EQ.10) GO TO 2000
      CALL RESTOR(DR,IBND,NN,NBNDRY,LOADS,NNMAX)
      CALL RESTOR(FR,IBND,NN,NBNDRY,LOADS,NNMAX)
      DO 112 I=1,NN
      DO 112 J=1,LOADS
  112 DR(I,J)=DR(I,J)/EEE
      DO 180 I = 1,LOADS
      ENGSTR(I) = 0.0
      DO 179 J = 1,NN
      ENGSTR(I) = ENGSTR(I) + FR(J,I)*DR(J,I)
  179 CONTINUE
      ENGSTR(I) = .5*ENGSTR(I)
  180 CONTINUE
      NPAGE=1
      LINES = 1
      DO 1501 I=1,LOADS
 1501 ENGTOT(I)=0.
      DO 300 L=1,MEMBS
      IF (NMAT .LE. 1) GO TO 85
      KX = MYOUNG(L)
      E = YOUNGM(KX)*10**6
      PMU = POISON(KX)
      E1 = E/EEE
      CALL ELSTIC(E1,PMU,EE)
      TYPE = NNODES(L)*10 + KX
      IF (MSSTRS .EQ. 0) GO TO 86
      KY = 3*(KX-1) + 1
      ALS(1) = ALSTRS(KY)
      ALS(2) = ALSTRS(KY+1)
      ALS(3) = ALSTRS(KY+2)
      GO TO 86
   85 TYPE = NNODES(L)*10 + 1
   86 IF((LINES+LOADS) .LT. 54 .AND. L .GT. 1)GO TO 84
      LINES=1
```

```
      WRITE(6,98)NPAGE
      NPAGE=NPAGE+1
      WRITE(6,83)
   84 CONTINUE
      CALL COORD(MA(L),MB(L),MC(L),MD(L),X,Y,Z,AA,XI,ETA,AL,NNODES(L),0)
      CALL ELFORC(AA,DR,EDR,MM,MA(L),MB(L),MC(L),MD(L),NNODES(L),LOADS,
     1NNMAX)
      IF(NNODES(L) .LE. 3)GO TO 110
      CALL QDRLTL(EK,EKK,TH(L),QUAD,MA(L),MB(L),MC(L),MD(L),MAA,MBB,MCC,
     1XI, ETA,NNODES(L),EE,TRANG,1)
      CALL QLSTRS(EDR,EDDR,XI,ETA,MAA,MBB,MCC,SX,SY,SXY,EFSTRS,E,PMU,
     1LOADS,SSX,SSY,SSXY,EFFSTR,KTR,EKK,ELEENG,SFTM,ALS,ESR,NNODES(L))
      KX=KTR(1)
      ELEENG(1)=ELEENG(1)*0.5*TH(L)
      ENGTOT(1)=ENGTOT(1)+ELEENG(1)
      IF (NNODES(L) .EQ. 5) GO TO 220
      WRITE(6,87) L,TH(L),QUAD,TYPE,MA(L),MB(L),MC(L),MD(L),SSX(KX,1),
     1SSY(KX,1),SSXY(KX,1),(EFFSTR(I,1),I=1,4),ELEENG(1),SFTM(1)
  222 IF(LOADS .EQ. 1)GO TO 300
      DO 211 K=2,LOADS
      KX=KTR(K)
      ELEENG(K)=ELEENG(K)*0.5*TH(L)
      ENGTOT(K)=ENGTOT(K)+ELEENG(K)
      IF (NNODES(L) .EQ. 5) GO TO 225
      WRITE(6,95)SSX(KX,K),SSY(KX,K),SSXY(KX,K),(EFFSTR (I,K),I=1,4),
     1ELEENG(K),SFTM(K)
      GO TO 211
  225 WRITE(6,82) SSXY(KX,K),(EFFSTR(I,K),I=1,4),ELEENG(K),SFTM(K)
  211 CONTINUE
      GO TO 300
  220 WRITE(6,81) L,TH(L),QUAD,TYPE,MA(L),MB(L),MC(L),MD(L),
     1SSXY(KX,1),(EFFSTR(I,1),I=1,4),ELEENG(1),SFTM(1)
      GO TO 222
  110 IF(NNODES(L) .LT. 3)GO TO 213
      CALL STRESS(EDR,XI,ETA,1,2,3,SX,SY,SXY,EFSTRS,E,PMU,ALS,ESR,
     1LOADS,ELEENG,TRIANG,3)
      ELEENG(1)=ELEENG(1)*0.5*TH(L)
      ENGTOT(1)=ENGTOT(1)+ELEENG(1)
      SFTM(1) = (1.0 - ESR(1))/ESR(1)
      WRITE(6,88) L,TH(L),TRIANG,TYPE,MA(L),MB(L),MC(L),SX(1),SY(1),
     1SXY(1),EFSTRS(1),ELEENG(1),SFTM(1)
      IF(LOADS .EQ. 1)GO TO 300
      DO 212 K=2,LOADS
      SFTM(K) = (1.0 - ESR(K))/ESR(K)
      ELEENG(K)=ELEENG(K)*0.5*TH(L)
      ENGTOT(K)=ENGTOT(K)+ELEENG(K)
  212 WRITE(6,94)SX(K),SY(K),SXY(K),EFSTRS(K),ELEENG(K),SFTM(K)
      GO TO 300
  213 DO 215 K=1,LOADS
      SX(K)=E*(EDR(2,K)-EDR(1,K))/AL
      ELEENG(K)=(0.5*SX(K)**2/E  )*AL*TH(L)
      ESF(K) = SQRT((SX(K)/ALS(1))**2)
      SFTM(K) = (1.0 - ESR(K))/ESR(K)
      IF (SX(K) .GE. 0.0) GO TO 215
      ESF(K) = SQRT((SX(K)/ALS(2))**2)
      SFTM(K) = (1.0 - ESR(K))/ESR(K)
  215 ENGTOT(K)=ENGTOT(K)+ELEENG(K)
```

```
      WRITE(6,89) L,TH(L),AL,TYPE,MA(L),MB(L),SX(1),ELEENG(1)          ANALYZE  287
      IF(LOADS .EQ. 1)GO TO 300                                        ANALYZE  288
      DO 214 K=2,LOADS                                                 ANALYZE  289
  214 WRITE(6,93)SX(K),ELEENG(K),SFTM(K)                               ANALYZE  290
  300 LINES=LINES+LOADS+1                                              ANALYZE  291
      DO 1503 KL=1,LOADS                                               ANALYZE  292
 1503 WRITE(6,1502)KL,ENGTOT(KL),ENGSTR(KL)                            ANALYZE  293
 1502 FORMAT(///,20X,39HTHE TOTAL ENERGY FOR LOADING CONDITION ,I2,4H IS ANALYZE  294
     1 ,E12.4,2X,3H(U),10X,E12.4,3H(W))                                ANALYZE  295
   90 LINES=1                                                          ANALYZE  296
      CALL PRNTDR(FR,DF,X,Y,Z,NN,MM,LOADS,JOINTS,NPAGE,NNMAX)          ANALYZE  297
   83 FORMAT(1X,4HMEMB,2X,5HTHICK,3X,4HAREA,2X,4HTYPE,1X,2HMA,2X,2HMB, ANALYZE  298
     12X,2HMC,2X,2HMD,3X,7HSIGMA-X,4X,7HSIGMA-Y,3X,8HSIGMA-XY,3X,      ANALYZE  299
     27HEFSTR-1,3X,7HEFSTR-2,3X,7HEFSTR-3,3X,7HEFSTR-4,4X,6HENERGY,    ANALYZE  300
     36X,2HMS)                                                         ANALYZE  301
   81 FORMAT(/I5,F7.3,F9.2,5I4,22X,E11.4,5E10.4,E10.3)                 ANALYZE  302
   82 FORMAT(63X,E11.4,5E10.4,E10.3)                                   ANALYZE  303
   87 FORMAT(/I5, F7.3,F9.2,5I4,3E11.4,5E10.4,E10.3)                   ANALYZE  304
   88 FORMAT(/I5, F7.3,F9.2,4I4,4X,3E11.4,E10.4,30X,E10.4,E10.3)       ANALYZE  305
   89 FORMAT(/I5, F7.3,F9.2,3I4,8X,E11.4,62X,E10.4,E10.3)              ANALYZE  306
   93 FORMAT(41X,E11.4,62X,E10.4,E10.3)                                ANALYZE  307
   94 FORMAT(41X,3E11.4,E10.4,30X,E10.4,E10.3)                         ANALYZE  308
   95 FORMAT(41X,3E11.4,5E10.4,E10.3)                                  ANALYZE  309
   98 FORMAT(1H1,120X,5HPAGE ,I3/)                                     ANALYZE  310
 2000 IF(KSTR.EQ.NSTR) GO TO 1000                                      ANALYZE  311
      KSTR=KSTR+1                                                      ANALYZE  312
      GO TO 1                                                          ANALYZE  313
 1000 CONTINUE                                                         ANALYZE  314
      STOP                                                             ANALYZE  315
      END                                                              ANALYZE  316
```

```
      SUBROUTINE POP(MMB,JN,MM,MA,MB,MC,MD,KTYPE,IC,ID,NZ,NR)
      DIMENSION MA(1),MB(1),MC(1),MD(1),IC(1),ID(1),KTYPE(1)
      IX(I,J)=I*(J-1)+1
      NZ=0
      NN=MM*JN
      NET=0
      DO 10 I=1,NN
   10 IC(I)=NN
      DO 50 L=1,MMB
      NNODES=2
      ITRI=0
      KX=IX(MM,MA(L))
      KY=IX(MM,MB(L))
   15 IF(IC(KY) .LT. KX) GO TO 18
      DO 19 I=1,MM
      IC(KY)=KX
   19 KY=KY+1
   18 IF(KTYPE(L)-3) 20,16,17
   16 IF(ITRI .EQ. 1)GO TO 20
      KY=IX(MM,MC(L))
      ITRI=1
      NNODES=3
      GO TO 15
   17 IF(ITRI .EQ. 2)GO TO 20
      IF(ITRI .EQ. 1)GO TO 14
      KY=IX(MM,MC(L))
      ITRI=ITRI+1
      NNODES=4
      GO TO 15
   14 KY=IX(MM,MD(L))
      ITRI=ITRI+1
      GO TO 15
   20 NET=NET+(MM**2)*((NNODES*(NNODES-1))/2)
   50 CONTINUE
      NET=NET-(MM**2)*NR
      DO 30 I=1,NN,MM
      IF(IC(I) .LT. I)GO TO 30
      KX=I
      DO 25 J=1,MM
      IC(KX)=I
   25 KX=KX+1
   30 CONTINUE
      DO 40 I=1,NN
      NZ=NZ+(I-IC(I)+1)
   40 ID(I)=NZ
      KX=(NN*(NN+1))/2
      NET=NET+(MM*(MM+1)*JN)/2
      WRITE(6,2)
      WRITE(6,3) KX,NET,NZ
      WRITE(6,4)
      WRITE(6,5)(IC(I),I=1,NN)
      WRITE(6,6)
      WRITE(6,5)(ID(I),I=1,NN)
    2 FORMAT(1H1,////20X,16HGROSS POPULATION,4X,14HNET POPULATION,
     14X,19HAPPARENT POPULATION///)
    3 FORMAT(18X,I14,I18,I22//)
    4 FORMAT(//2X,36HSTARTING ROW NUMBERS FOR EACH COLUMN///)
    5 FORMAT(5X,10I12)
    6 FORMAT(//2X,61HNUMBERS OF DIAGONAL ELEMENTS IN SINGLE ARRAYSTIFFNE
     1SS MATRIX ///)
      RETURN
      END
```

```
      SUBROUTINE ELSTIC(E,PMU,EE)
      DIMENSION EE(3,3)
      PMU1 = 1.0 - PMU**2
      EE(1,1) = E/PMU1
      EE(2,1) = E*PMU/PMU1
      EE(3,1) = 0.0
      EE(2,2) = EE(1,1)
      EE(3,2) = 0.0
      EE(3,3) = E/(2.*(1.0 + PMU))
      DO 18 I = 1,2
      IP = I + 1
      DO 18 J = IP,3
   18 EE(I,J) = EE(J,I)
      RETURN
      END
```

```
      SUBROUTINE COORD(K1,K2,K3,K4,X,Y,Z,AA,XI,ETA,AL,NND,NO)
      DIMENSION X(1),Y(1),Z(1),AA(3,3),AB(3),XI(5),ETA(5)
      XCOMP=X(K2)-X(K1)
      YCOMP=Y(K2)-Y(K1)
      ZCOMP=Z(K2)-Z(K1)
      AL=SQRT(XCOMP**2+YCOMP**2+ZCOMP**2)
      AA(1,1)=XCOMP/AL
      AA(1,2)=YCOMP/AL
      AA(1,3)=ZCOMP/AL
      IF(NND .LT. 3)RETURN
      XCOMP=X(K3)-X(K1)
      YCOMP=Y(K3)-Y(K1)
      ZCOMP=Z(K3)-Z(K1)
      AL=SQRT(XCOMP**2+YCOMP**2+ZCOMP**2)
      AB(1)=XCOMP/AL
      AB(2)=YCOMP/AL
      AB(3)=ZCOMP/AL
      AL=SQRT((AA(1,2)*AB(3)-AA(1,3)*AB(2))**2+(AA(1,3)*AB(1)
     1-AA(1,1)*AB(3))**2+(AA(1,1)*AB(2)-AA(1,2)*AB(1))**2)
      AA(2,1)=((AA(1,3)**2)*AB(1)-AA(1,1)*AA(1,3)*AB(3)-AA(1,1)*
     1AA(1,2)*AB(2)+(AA(1,2)**2)*AB(1))/AL
      AA(2,2)=((AA(1,1)**2)*AB(2)-AA(1,1)*AA(1,2)*AB(1)-AA(1,2)*
     2AA(1,3)*AB(3)+(AA(1,3)**2)*AB(2))/AL
      AA(2,3)=((AA(1,2)**2)*AB(3)-AA(1,2)*AA(1,3)*AB(2)-AA(1,1)*
     3AA(1,3)*AB(1)+(AA(1,1)**2)*AB(3))/AL
      IF(NO .EQ. 1)RETURN
      XI(1)=0.0
      ETA(1)=0.0
      XI(2)=(X(K2)-X(K1))*AA(1,1)+(Y(K2)-Y(K1))*AA(1,2)+(Z(K2)-Z(K1))*AA
     1(1,3)
      ETA(2)=0.0
      XI(3)=(X(K3)-X(K1))*AA(1,1)+(Y(K3)-Y(K1))*AA(1,2)+(Z(K3)-Z(K1))*AA
     1(1,3)
      ETA(3)=(X(K3)-X(K1))*AA(2,1)+(Y(K3)-Y(K1))*AA(2,2)+(Z(K3)-Z(K1))*A
     1A(2,3)
      IF(NND .LE. 3)RETURN
      XI(4)=(X(K4)-X(K1))*AA(1,1)+(Y(K4)-Y(K1))*AA(1,2)+(Z(K4)-Z(K1))*AA
     1(1,3)
      ETA(4)=(X(K4)-X(K1))*AA(2,1)+(Y(K4)-Y(K1))*AA(2,2)+(Z(K4)-Z(K1))*A
     1A(2,3)
      XI(5)=(XI(2)+XI(3)+XI(4)) /4.0
      ETA(5)=(ETA(3)+ETA(4))/4.0
      RETURN
      END
```

```
      SUBROUTINE ELSTIF(A,B,C,AE,MM,AL,E)
      DIMENSION A(3,3),B(12,12),C(12,12)
      EK=AE*E/AL
      DO 25 I=1,MM
      J=I+MM
      B(1,I)=EK*A(1,I)
      B(1,J)= -B(1,I)
      B(2,I) =-B(1,I)
   25 B(2,J)=B(1,I)
      DO 26 I=1,MM
      DO 26 J=1,MM
   26 C(I,J)=A(1,I)*B(1,J)
      DO 36 I=1,MM
      I1=I+MM
      DO 36 J=1,MM
      J1=J+MM
      C(I,J1)=-C(I,J)
      C(J1,I)=-C(I,J)
   36 C(I1,J1)=C(I,J)
      RETURN
      END
```

```
      SUBROUTINE PLSTIF(EKK,TH,TRIANG,MA,MB,MC,X,Y,EE,SHR,NONORM)
      DIMENSION EKK(12,12),X(1),Y(1),EE(3,3),
     1  U(6),            A(3,3),E1(3),E2(3),AX(3)
      CALL CRAMER(A,TRIANG,X,Y,MA,MB,MC)
      DO 20 I = 1,6
      DO 15 II = 1,6
   15 U(II) = 0.0
      U(I) = 1.0
      E1(1) = A(1,1)*U(1) + A(1,2)*U(3) + A(1,3)*U(5)
      E1(2) = A(2,1)*U(2) + A(2,2)*U(4) + A(2,3)*U(6)
      E1(3) = A(1,1)*U(2) + A(1,2)*U(4) + A(1,3)*U(6) + A(2,1)*U(1) +
     1        A(2,2)*U(3) + A(2,3)*U(5)
      DO 20 J = I,6
      DO 16 II = 1,6
   16 U(II) = 0.0
      U(J) = 1.0
      E2(1) = A(1,1)*U(1) + A(1,2)*U(3) + A(1,3)*U(5)
      E2(2) = A(2,1)*U(2) + A(2,2)*U(4) + A(2,3)*U(6)
      E2(3) = A(1,1)*U(2) + A(1,2)*U(4) + A(1,3)*U(6) + A(2,1)*U(1) +
     1        A(2,2)*U(3) + A(2,3)*U(5)
      EKK(I,J) = 0.0
      IF (NONORM .EQ. 0) GO TO 14
      AX(1)=SHR**2
      AX(2)=2.*AX(1)-1.
      AX(1)=2.*SQRT((1.-AX(1))*AX(1))
      E1(3)=(E1(2)-E1(1))*AX(1)+E1(3)*AX(2)
      E2(3)=(E2(2)-E2(1))*AX(1)+E2(3)*AX(2)
      E1(1) = 0.0
      E1(2) = 0.0
      E2(1) = 0.0
      E2(2) = 0.0
   14 DO 18 K = 1,3
      AX(K) = 0.0
      DO 17 L = 1,3
   17 AX(K) = AX(K) + EE(K,L)*E2(L)
   18 CONTINUE
      DO 19 K = 1,3
   19 EKK(I,J) = EKK(I,J) + E1(K)*AX(K)
      EKK(I,J) = EKK(I,J)*TH*TRIANG
   20 CONTINUE
      DO 30 I = 1,5
      IX = I + 1
      DO 30 J = IX,6
   30 EKK(J,I) = EKK(I,J)
      RETURN
      END
```

```
      SUBROUTINE CRAMER(A,TRIANG,X,Y,MA,MB,MC)
      DIMENSION A(3,3),X(1),Y(1)
      TRIANG = X(MA)*(Y(MB) - Y(MC)) -Y(MA)*(X(MB) - X(MC)) +
     1        (X(MB)*Y(MC) - X(MC)*Y(MB))
      A(1,1) = Y(MB) - Y(MC)
      A(2,1) = X(MC) - X(MB)
      A(3,1) = X(MB)*Y(MC) - X(MC)*Y(MB)
      A(1,2) = Y(MC) - Y(MA)
      A(2,2) = X(MA) - X(MC)
      A(3,2) = X(MC)*Y(MA) - X(MA)*Y(MC)
      A(1,3) = Y(MA) - Y(MB)
      A(2,3) = X(MB) - X(MA)
      A(3,3) = X(MA)*Y(MB) - X(MB)*Y(MA)
      DO 10 I = 1,3
      DO 10 J = 1,3
   10 A(I,J) = A(I,J)/TRIANG
      TRIANG =(ABS(TRIANG))/2.0
      RETURN
      END
```

```
      SUBROUTINE QDRLTL(EK,EKK,TH,QUAD,MA,MB,MC,MD,MAA,MBB,MCC,XI,ETA,
     1NNODES,EE,TRANG,NO)
      DIMENSION EK(12,12),EKK(12,12),MAA(1),MBB(1),MCC(1),XI(5),ETA(5)
     1,EE(3,3),TRANG(1)
      DO 125 I=1,12
      DO 125 J=1,12
  125 EK(I,J)=0.
      NNRM=0
      SHR=1.0
      IF(NNODES     .LE. 4)GO TO 108
      NNRM=1
      IF(NNODES     .EQ. 5)GO TO 108
      IF(NNODES     - 7)104,105,106
  104 XCOMP=XI(3)-XI(2)
      YCOMP=ETA(3)-ETA(2)
      GO TO 107
  105 XCOMP=XI(4)-XI(3)
      YCOMP=ETA(4)-ETA(3)
      GO TO 107
  106 XCOMP=XI(4)-XI(1)
      YCOMP=ETA(4)-ETA(1)
  107 ALL=SQRT(XCOMP**2+YCOMP**2)
      SHR=XCOMP/ALL
  108 QUAD=0.
      DO 130 I=1,4
      CALL PLSTIF(EKK,TH,TRIANG,MAA(I),MBB(I),MCC(I),XI,ETA,EE,SHR,NNRM)
      QUAD=QUAD+TRIANG
      TRANG(I)=TRIANG
  130 CALL SUM(EK,EKK,MAA(I),MBB(I),MCC(I))
      CALL CONDNS(EK,EKK,MA,MB,MC,MD,NO)
      RETURN
      END
```

```
SUBROUTINE SUM        74/74    OPT=1    FTN 4.6+446     08/21/78  10.12.39     PAGE    1

          SUBROUTINE SUM(EK,EKK,MA,MB,MC)
          DIMENSION EK(12,12),EKK(12,12),NA(3)
          M=2
          NA(1)=2*(MA-1)+1
          NA(2)=2*(MB-1)+1
          NA(3)=2*(MC-1)+1
          IH=0
          DO 100 I=1,6
          JH=0
          IF(I .LE. IH)GO TO 30
          IH=IH+M
          IHH=IH/M
          KX=NA(IHH)
       30 DO 90 J=1,6
          IF(J .LE. JH)GO TO 60
          JH=JH+M
          IHH=JH/M
          KY=NA(IHH)
       60 EK(KX,KY)=EK(KX,KY)+EKK(I,J)
       90 KY=KY+1
      100 KX=KX+1
          RETURN
          END
```

```
      SUBROUTINE CONDNS(EK,EKK,MA,MB,MC,MD,NO)
      DIMENSION EK(12,12),EKK(12,12)
      DO 5 I=1,8
      DO 5 J=1,8
    5 EKK(I,J)=0.
      DET=EK(9,9)*EK(10,10)-EK(9,10)**2
      AX=EK(9,9)
      EK(9,9)=EK(10,10)/DET
      EK(10,10)=AX/DET
      EK(9,10)=-EK(9,10)/DET
      EK(10,9)=EK(9,10)
      KX=0
      DO 10 I=9,10
      KX=KX+1
      DO 10 J=1,8
      DO 10 K=9,10
   10 EKK(KX,J)=EKK(KX,J)+EK(I,K)*EK(K,J)
      IF(NO .EQ. 1)RETURN
      KX=0
      DO 20 I=9,10
      KX=KX+1
      DO 20 J=1,8
      EK(I,J)=EKK(KX,J)
   20 EKK(KX,J)=0
      DO 30 I=1,8
      DO 30 J=1,8
      DO 30 K=9,10
   30 EKK(I,J)=EKK(I,J)+EK(I,K)*EK(K,J)
      DO 40 I=1,8
      DO 40 J=1,8
   40 EK(I,J)=EK(I,J)-EKK(I,J)
      IF(MC .LT. MB)CALL CHANGE(EK,3,5,4,12,12,0)
      IF(MD .LT. MB)CALL CHANGE(EK,3,7,4,12,12,0)
      IF(MD .LT. MC)CALL CHANGE(EK,5,7,4,12,12,0)
      RETURN
      END
```

```
      SUBROUTINE CHANGE(EK,IX,IY,NND,M,L,IR)
      DIMENSION EK( M, L)
      KX=IX
      KY=IY
      M2=2*NND
      IF(IR .EQ. 1)M2=L
      DO 10 I=1,2
      DO 5 J=1,M2
      AX=EK(KX,J)
      EK(KX,J)=EK(KY,J)
    5 EK(KY,J)=AX
      KX=KX+1
   10 KY=KY+1
      IF(IR .EQ. 1)RETURN
      KX=KX-2
      KY=KY-2
      DO 20 I=1,2
      DO 15 J=1,M2
      AX=EK(J,KX)
      EK(J,KX)=EK(J,KY)
   15 EK(J,KY)=AX
      KX=KX+1
   20 KY=KY+1
      RETURN
      END
```

```
      SUBROUTINE TRNSFM(EK,AA,B,C,MM,NND,M)
      DIMENSION EK(12,12),AA(3,3),B( M, M),C( M, M)
      M2=2*NND
      IF(NND .GT. 4)M2=8
      M3=MM*NND
      IF(NND .GT. 4)M3=4*MM
      DO 100 I=1,M2
      JA=MM
      KA=0
      IA=0
      DO 100 J=1,M3
      B(I,J)=0.0
      IF(J-JA)90,90,80
   80 JA=JA+MM
      KA=KA+2
      IA=IA+MM
   90 JAA=J-IA
      DO 100 K=1,2
      KAA=K+KA
  100 B(I,J)=B(I,J)+EK(I,KAA)*AA(K,JAA)
      DO 200 J=1,M3
      JA=MM
      KA=0
      IA=0
      DO 200 I=1,M3
      C(I,J)=0.0
      IF(I-JA)190,190,180
  180 JA=JA+MM
      KA=KA+2
      IA=IA+MM
  190 JAA=I-IA
      DO 200 K=1,2
      KAA=K+KA
  200 C(I,J)=C(I,J)+AA(K,JAA)*B(KAA,J)
      RETURN
      END
```

```
      SUBROUTINE ASEMBL(A,B,MA,MB,MC,MD,MM,ID,NNODES,M)
      DIMENSION A(1),B(M,M),ID(1),NA(4),NAA(3)
      IX(I,J)=I*(J-1)+1
      NND=NNODES
      IF(NND .GT. 4)NND=4
      M2=NND*MM
      NA(1)=IX(MM,MA)
      NA(2)=IX(MM,MB)
      IF(NNODES .GE. 3)NA(3)=IX(MM,MC)
      IF(NNODES .GE. 4)NA(4)=IX(MM,MD)
      IF(NNODES .LE. 3)GO TO 5
      DO 4 I=1,3
      KX=I/3
      KY=I/2
      IF(NA(KX+2) .LT. NA(KY+3))GO TO 4
      KH=NA(KX+2)
      NA(KX+2)=NA(KY+3)
      NA(KY+3)=KH
    4 CONTINUE
    5 DO 10 I=2,NND
   10 NAA(I-1)=NA(I)-NA(I-1)-MM
      KH=MM
      IAA=NA(1)
      KHH=1
      DO 30 J=1,M2
      IF(J .LE. KH)GO TO 15
      KHH=KHH+1
      IAA=NA(KHH)
      KH=KH+MM
   15 JX=ID(IAA)-IAA+NA(1)
      KY=MM
      DO 25 I=1,J
      IF(J .LE.KY .OR. I .LE. KY)GO TO 20
      KX=I/MM
      JX=JX+NAA(KX)
      KY=KY+MM
   20 A(JX)=A(JX)+B(I,J)
   25 JX=JX+1
   30 IAA=IAA+1
      RETURN
      END
```

```
      SUBROUTINE PRINTK(SK,IDIAG,NN)
      DIMENSION SK(1),IDIAG(1)
      DO 80 I=1,NN
      IF(I .GT. 1) GO TO 65
      KX=1
      KY=1
      GO TO 70
   65 KX=IDIAG(I-1)+1
      KY=IDIAG(I)
   70 WRITE(6, 3) I
   80 WRITE(6,2) (SK(K),K=KX,KY)
    3 FORMAT(I4)
    2 FORMAT(10X,10E12.4)
      RETURN
      END
```

```
      SUBROUTINE BOUND2(A,IB,N,NB,ID,IC)
      DIMENSION A(1),IB(1),ID(1),IC(1)
      IH=NB
      NH=N
      DO 30 JA=1,NB
      IA=IB(IH)
      IF(IA .GE. NH) GO TO 20
      KH=IA+1
      IF(IA .GT. 1) GO TO 5
      KX=1
      JX=1
      GO TO 6
    5 JX=ID(IA)-ID(IA-1)
      KX=ID(IA-1)+1
    6 DO 10 I=KH,NH
      KY=1
      IF(IC(I) .LE. IA) GO TO 7
      IC(I-1)=IC(I)-1
      I1=I
      KY=0
      GO TO 8
    7 IC(I-1)=IC(I)
      I1=I-1
    8 K=IC(I)
      ID(I-1)=ID(I)-JX-KY
      DO 10 J=K,I1
      IF(J .EQ. IA) JX=JX+1
      KXX=KX+JX
      A(KX)=A(KXX)
   10 KX=KX+1
   20 NH=NH-1
      IH=IH-1
   30 CONTINUE
      PETURN
      END
```

```
      SUBROUTINE REDUCE (F,IB,N,NB,L,NN)
      DIMENSION F(NN,L) ,IB(1)
      DO 5 J=1,L
      IH=NB
      NH=N
    1 I=IB(IH)
      IF(I-NH) 2,4,4
    2 NH1=NH-1
      DO 3 K=I,NH1
      K1=K+1
    3 F(K,J) =F(K1,J)
    4 IH=IH-1
      NH=NH-1
      IF(IH.EQ.0) GO TO 5
      GO TO1
    5 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE  GAUSS(A,F,D,IC,ID,L,N,NN,NDCOMP)
      DIMENSION A(1),IC(1), ID(1),F(NN,L),D(NN,L)
      IF(NDCOMP .EQ. 1)GO TO 15
      DO 10 I=1,N
      I1=I-1
      DO  9 J=I,N
      IF(IC(J) .GT. I)GO TO 9
      IX=ID(J)-J+I
      IF(I1 .EQ. 0)GO TO 8
      DO  7 K=1,I1
      IF(IC(J) .GT. K .OR. IC(I) .GT. K)GO TO 7
      KX=ID(I)-I+K
      KY=ID(J)-J+K
      KZ=ID(K)
      A(IX)=A(IX)-(A(KX)*A(KZ)* A(KY))
    7 CONTINUE
    8 IF(I .EQ. J)GO TO 9
      KZ=ID(I)
      IF(A(KZ))5,6,5
    5 A(IX)=A(IX)/A(KZ)
    9 CONTINUE
   10 CONTINUE
   15 DO 40 K=1,L
      DO 30 I=1,N
      D(I,K)=F(I,K)
      I1=I-1
      IF(I1 .EQ. 0) GO TO 30
      DO 20 J=1,I1
      IF(IC(I) .GT. J)GO TO 20
      IX=ID(I)-I+J
      D(I,K)=D(I,K)-A(IX)*D(J,K)
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
      DO 70 I=1,N
      KX=ID(I)
      DO 70 K=1,L
   70 D(I,K)=D(I,K)/A(KX)
      DO 90 K=1,L
      IX=N
      DO 90 I=2,N
      IX=IX-1
      I1=I-1
      KX=IX
      DO 80 J=1,I1
      KX=KX+1
      IF(IC(KX) .GT. IX)GO TO 80
      KY=ID(KX)-KX+IX
      D(IX,K)=D(IX,K)-A(KY)*D(KX,K)
   80 CONTINUE
   90 CONTINUE
  110 RETURN
    6 NDCOMP=10
      WRITE(6,120)I
  120 FORMAT(///2X,46HSTRUCTURE IS UNSTABLE, THE DEGREE OF FREEDOM =,I5)
      RETURN
      END
```

```
      SUBROUTINE RESTOR( D,IB,N,NB,L,NN)
      DIMENSION D(NN,L),IB(1),TDR1(10),TDR2(10)
      NH=N-NB
      IH=1
    1 I=IB(IH)
      IF(I.GT.NH) GO TO 7
      DO 2 K=1,L
      TDR1(K)=D(I,K)
    2 D(I,K)=0.
    3 J=I+1
      IF(J.GT.NH) GO TO 5
      DO 4 K=1,L
    4 TDR2(K)=D(J,K)
    5 DO 6 K=1,L
      D(J,K) =TDR1(K)
    6 TDR1(K)=TDR2(K)
      IF(I.GE.NH) GO TO 9
      I=I+1
      GO TO 3
    7 DO 8 K=1,L
    8 D(I,K)=0.
    9 IF(IH.GE.NB) GO TO 10
      IH=IH+1
      NH=NH+1
      GO TO 1
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ELFORC(AA,DR,EDR,MM,MA,MB,MC,MD,NNODES,LOADS,NN)
      DIMENSION AA(3,3),DR(NN,LOADS),EDR(12,LOADS),NCON(4)
      NCON(1)=MM*(MA    -1)+1
      NCON(2)=MM*(MB    -1)+1
      IF(NNODES    .GE. 3)NCON(3)=MM*(MC    -1)+1
      IF(NNODES    .GE. 4)NCON(4)=MM*(MD    -1)+1
      NND=NNODES
      IF(NND .GT. 4)NND=4
      NDSP=1
      IF(NND .GT. 2)NDSP=2
      DO 86 K=1,LOADS
      KH=1
      DO 86 KK=1,NND
      DO 86 I=1,NDSP
      KX=NCON (KK)
      EDR(KH,K)=0
      DO 85 J=1,MM
      EDR(KH,K)=EDR(KH,K)+AA(I,J)*DR(KX,K)
   85 KX=KX+1
   86 KH=KH+1
      RETURN
      END
```

```
      SUBROUTINE STRESS(UV,X,Y,MA,MB,MC,SX,SY,SXY,EFST,E,P,ALS,ESR,
     1               L,ENG,TRIANG,NND)
      DIMENSION UV(12,L),X(1),Y(1),SX(1),SY(1),SXY(1),EX(10),EY(10),
     1EXY(10),A(3,3)       ,EFST(1),ENG(1),ALS(3),ESR(1)
      CALL CRAMER(A,TRIANG,X,Y,MA,MB,MC)
      DO 30 K=1,L
      EX(K)=0.
      EY(K)=0.
      EXY(K)=0.
      KX=0
      DO 20 I=1,3
      IX=I +KX
      EX(K)=EX(K)+A(1,I)*UV(IX,K)
      EY(K)=EY(K)+A(2,I)*UV(IX+1,K)
      EXY(K)=EXY(K)+A(2,I)*UV(IX,K)+A(1,I)*UV(IX+1,K)
   20 KX=KX+1
   30 CONTINUE
      EMU=E/(1.0-P**2)
      G=(0.5*E)/(1.0+P)
      DO 40 K=1,L
      SX(K)=(EX(K)+P*EY(K))*EMU
      SY(K)=(P*EX(K)+EY(K))*EMU
   40 SXY(K)=G*EXY(K)
      DO 90 K=1,L
      AAX = ALS(1)
      AAY = ALS(1)
      AAXY = ALS(3)
      IF (SX(K) .LT. 0.0) AAX = ALS(2)
      IF (SY(K) .LT. 0.0) AAY = ALS(2)
      EFST(K)=SQRT(SX(K)**2+SY(K)**2-SX(K)*SY(K)+3.*(SXY(K)**2))
      ENG(K)=(SX(K)*EX(K)+SY(K)*EY(K)+SXY(K)*EXY(K))*TRIANG
      IF(NND .GT. 4)ENG(K)=(SXY(K)*EXY(K))*TRIANG
      ESR(K) = SQRT((SX(K)/AAX)**2 + (SY(K)/AAY)**2
     1 - ((SX(K)*SY(K))/(AAX*AAY)) + (SXY(K)/AAXY)**2)
      IF (NND .GT. 4) ESR(K) = ABS(SXY(K))/AAXY
   90 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE QLSTRS(EDR,EDDR,XI,ETA,MAA,MBB,MCC,SX,SY,SXY,EFSTRS,E,
     1PMU,LOADS,SSX,SSY,SSXY,EFFSTR,KTR,EKK,ENG,SFTM,ALS,ESR,NND)
      DIMENSION EDR(12,LOADS),EDDR(12,LOADS),XI(1),ETA(1),MAA(1),MBB(1),
     1MCC(1),SX(1),SY(1),SXY(1),EFSTRS(1),SSX(4,LOADS),SSY(4,LOADS),
     2SSXY(4,LOADS),EFFSTR(4,LOADS),KTR(1),EKK(12,12),ENG(1),ENGG(10)
     3,ESR(1),SFTM(1),ALS(1)
      DO 115 K=1,LOADS
      SFTM(K) = 0.0
      ENG(K)=0.
      KX=0
      DO 115 I=9,10
      KX=KX+1
      EDR(I,K)=0.
      DO 114 J=1,8
  114 EDR(I,K)=EDR(I,K)+EKK(KX,J)*EDR(J,K)
      EDR(I,K)=-EDR(I,K)
  115 CONTINUE
      DO 116 K=1,LOADS
      EDDR(5,K)=EDR(9,K)
  116 EDDR(6,K)=EDR(10,K)
      KX=1
      KY=3
      QUAD = 0.0
      DO 200 I=1,4
      IF(I .LT. 4)GO TO 117
      KX=1
      KY=7
  117 DO 119 J=1,2
      DO 118 K=1,LOADS
      EDDR(J,K)=EDR(KX,K)
  118 EDDR(J+2,K)=EDR(KY,K)
      KX=KX+1
  119 KY=KY+1
      CALL STRESS(EDDR,XI,ETA,MAA(I),MBB(I),MCC(I),SX,SY,SXY,EFSTRS,
     1         E,PMU,ALS,ESR,LOADS,ENGG,TRIANG,NND)
      QUAD = QUAD + TRIANG
      DO 201 J=1,LOADS
      ENG(J)=ENG(J)+ENGG(J)
      SSX(I,J)=SX(J)
      SSY(I,J)=SY(J)
      SSXY(I,J)=SXY(J)
      EFFSTR(I,J)=EFSTRS(J)
      IF(NND .GT. 4)EFFSTR(I,J)=ABS(SXY(J))
      SFTM(J) = SFTM(J) + ESR(J)*TRIANG
  201 CONTINUE
  200 CONTINUE
      DO 205 J=1,LOADS
      SFTM(J) = SFTM(J)/QUAD
      SFTM(J) = (1.0 - SFTM(J))/SFTM(J)
      AMAX=0.
      DO 204 I=1,4
      IF(AMAX .GT. EFFSTR(I,J))GO TO 204
      AMAX=EFFSTR(I,J)
      KTR(J)=I
  204 CONTINUE
  205 CONTINUE
      DO 210 J=1,LOADS
      KX=KTR(J)
      AMAX=EFFSTR(KX,J)
      DO 209 I=1,4
  209 EFFSTR(I,J)=EFFSTR(I,J)/AMAX
      EFFSTR(KX,J)=AMAX
  210 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PRNTOR(A,B,X,Y,Z,N,M,L,NJ,NP,NN)
      DIMENSION A(NN,L),B(NN,L),X(1),Y(1),Z(1)
      NP=NP+1
      LINES=1
      WRITE(6,1)NP
      WRITE(6,2)
      DO 10 I=1,NJ
      IF (LINES+L-54)4,3,3
    3 LINES=1
      WRITE(6, 1) NP
      WRITE(6, 2)
      NP=NP+1
    4 KH=M*I
      KHH=KH-M+1
      IF(M .LT. 3)GO TO 11
      WRITE(6, 9)I,X(I),Y(I),Z(I),( A(J,1),J=KHH,KH),( B(J,1),J=KHH,KH)
      GO TO 12
   11 WRITE(6, 5)I,X(I),Y(I), ( A(J,1),J=KHH,KH),( B(J,1),J=KHH,KH)
   12 IF(L .EQ. 1) GOTO 8
      DO 7   K=2,L
      IF(M .LT. 3)GO TO 13
      WRITE (6,  6) ( A(J,K) ,J=KHH,KH), ( B(J,K), J=KHH,KH)
      GO TO 7
   13 WRITE (6, 15) ( A(J,K) ,J=KHH,KH), ( B(J,K), J=KHH,KH)
    7 CONTINUE
    8 LINES =LINES +L+1
      IF(L     .EQ. 1)LINES=LINES-1
   10 CONTINUE
    1 FORMAT(1H1,120X,5HPAGE ,I3/)
    2 FORMAT(  1X,5HJOINT,8X,2H-X,8X,2H-Y,8X,2H-Z,8X,7HFORCE-X,
     17X,7HFORCE-Y,7X,7HFORCE-Z,8X,7HDISPL-X,10X,7HDISPL-Y,10X,
     27HDISPL-Z//)
    9 FORMAT(/I5,F14.3,F10.3,F10.3,F12.3,F14.3,F14.3,1PE18.8,
     11PE17.8,1PE17.8)
    5 FORMAT(/I5,F14.3,F10.3,10X,F12.3,F14.3,14X,1PE18.8,1PE17.8)
    6 FORMAT(39X,F12.3,F14.3,F14.3,1PE18.8,1PE17.8,1PE17.8)
   15 FORMAT(39X,F12.3,F14.3,14X,1PE18.8,1PE17.8)
      RETURN
      END
```

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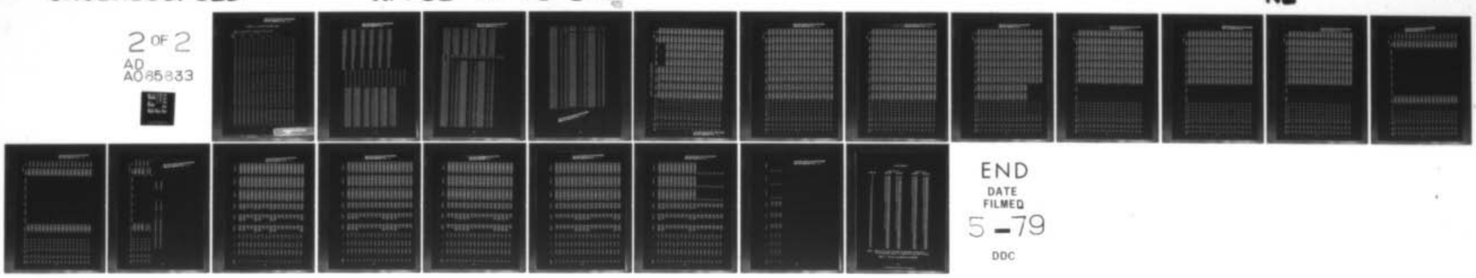
'ANALYZE' - ANALYSIS OF AEROSPACE STRUCTURES WITH MEMBRANE ELEM--ETC(U)
DEC 78 V B VENKAYYA, V A TISCHLER

UNCLASSIFIED AFFDL-TR-78-170 NL

2 OF 2
AD
A065633



END
DATE
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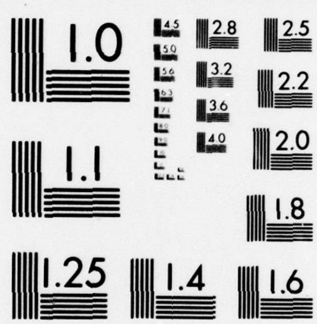
5-79
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

APPENDIX C: LISTING OF THE SAMPLE DATA

```
     1
   ANALYZE  DEMO PROBLEM--INTERMEDIATE COMPLEXITY WING
  158    58    30     2     3    10     1     0     2     1
10.5         .3           .1
60.         60.          35.          60.          60.          35.
10.5         .3           .1          10.5         .3           .1
     3     3     4     4     4     4     4     4     4     4     4     4     4     4
     4     4     4     4     4     4     4     4     4     4     4     4     4     4
     4     4     4     4     4     4     4     4     4     4     4     4     4     4
     4     4     4     4     4     4     4     4     4     4     4     4     4     4
     4     4     4     4     4     4     4     4     5     5     5     5     5     5
     5     5     5     5     5     5     5     5     5     5     5     5     5     5
     5     5     5     5     5     5     5     5     5     5     5     5     5     5
     5     5     5     5     5     5     5     5     5     5     5     5     5     5
     5     5     5     5     5     5     5     2     2     2     2     2     2     2
     2     2     2     2     2     2     2     2     2     2     2     2     2     2
     2     2     2     2     2     2     2     2     2     2     2     2     2     2
     2     2     2     2
     1     2     3     4     5     6     7     8     1     2    11    12    13    14
    15    16    19    20    21    22    23    24    25    26    29    30    31    32
    33    34    35    36    39    40    41    42    43    44    45    46    49    50
    51    52    53    54    55    56    59    60    61    62    63    64    65    66
    69    70    71    72    73    74    75    76     1     3     5     7     1    11
    13    15    19    21    23    25    29    31    33    35    39    41    43    45
    49    51    53    55    59    61    63    65    69    71    73    75     1    19
    29    39    49    59    69     5    13    23    33    43    53    63    73     9
    17    27    37    47    57    67    77     1     3     5     7     9    11    13
    15    17    19    21    23    25    27    29    31    33    35    37    39    41
    43    45    47    49    51    53    55    57    59    61    63    65    67    69
    71    73    75    77
     3     4     5     6     7     8     9    10    11    12    13    14    15    16
    17    18    21    22    23    24    25    26    27    28    31    32    33    34
    35    36    37    38    41    42    43    44    45    46    47    48    51    52
    53    54    55    56    57    58    61    62    63    64    65    66    67    68
    71    72    73    74    75    76    77    78     2     4     6     8     2    12
    14    16    20    22    24    26    30    32    34    36    40    42    44    46
    50    52    54    56    60    62    64    66    70    72    74    76     2    20
    30    40    50    60    70     6    14    24    34    44    54    64    74    10
    18    28    38    48    58    68    78     2     4     6     8    10    12    14
    16    18    20    22    24    26    28    30    32    34    36    38    40    42
    44    46    48    50    52    54    56    58    60    62    64    66    68    70
    72    74    76    78
    11    12    13    14    15    16    17    18    21    22    23    24    25    26
    27    28    31    32    33    34    35    36    37    38    41    42    43    44
    45    46    47    48    51    52    53    54    55    56    57    58    61    62
    63    64    65    66    67    68    71    72    73    74    75    76    77    78
    81    82    83    84    85    86    87    88     4     6     8    10    12    14
    16    18    22    24    26    28    32    34    36    38    42    44    46    48
    52    54    56    58    62    64    66    68    72    74    76    78    20    30
    40    50    60    70    80    14    24    34    44    54    64    74    84    18
    28    38    48    58    68    78    88     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0
     0     0    11    12    13    14    15    16    19    20    21    22    23    24
    25    26    29    30    31    32    33    34    35    36    39    40    41    42
    43    44    45    46    49    50    51    52    53    54    55    56    59    60
    61    62    63    64    65    66    69    70    71    72    73    74    75    76
    79    80    81    82    83    84    85    86     3     5     7     9    11    13
    15    17    21    23    25    27    31    33    35    37    41    43    45    47
    51    53    55    57    61    63    65    67    71    73    75    77    19    29
    39    49    59    69    79    13    23    33    43    53    63    73    83    17
    27    37    47    57    67    77    87     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0     0     0
     0     0     0     0     0     0     0     0     0     0     0     0     0     0
```

0 0/ 0 0

.0312	.0312	.0312	.0312	.0312	.0312
.0416	.0416	.0312	.0312	.0312	.0312
.0364	.0364	.0364	.0364	.0364	.0364
.0468	.0468	.0468	.0468	.0520	.0520
.0416	.0416	.0624	.0624	.0624	.0624
.0624	.0624	.0468	.0468	.0676	.0676
.0728	.0728	.0780	.0780	.0520	.0520
.0728	.0728	.0936	.0936	.1040	.1040
.0520	.0520	.0780	.0780	.1144	.1144
.1196	.1196	.0468	.0468	.0884	.0884
.1248	.1248	.1404	.1404	.019	.019
.019	.019	.019	.019	.019	.019
.019	.019	.019	.019	.019	.019
.019	.019	.019	.019	.019	.019
.019	.019	.019	.019	.022	.021
.019	.019	.025	.024	.019	.019
.019	.038	.042	.048	.047	.039
.031	.026	.037	.050	.058	.065
.073	.101	.126	.019	.039	.044
.053	.065	.079	.092	.100	.02
.02	.02	.02	.02	.02	.02
.02	.02	.02	.02	.02	.02
.02	.02	.02	.02	.02	.02
.02	.02	.02	.02	.02	.02
.02	.02	.02	.02	.02	.02
.02	.02	.02	.02	.02	.02
.02	.02				

2	2	2	2	2	2	2	2	2	2	2	2	2	2
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2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	1	1	1	1	1	1
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1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1										

63.5000	90.0000	1.1250	63.5000	90.0000	-1.1250
70.8330	90.0000	1.3130	70.8330	90.0000	-1.3130
78.1670	90.0000	1.5000	78.1670	90.0000	-1.5000
85.5000	90.0000	1.3130	85.5000	90.0000	-1.3130
92.8330	90.0000	1.1250	92.8330	90.0000	-1.1250
69.6860	87.4710	1.3490	69.6860	87.4710	-1.3490
76.0970	84.8510	1.5860	76.0970	84.8510	-1.5860
82.7460	82.1330	1.4270	82.7460	82.1330	-1.4270
89.6470	79.3120	1.2590	89.6470	79.3120	-1.2590
57.2660	77.6690	1.2790	57.2660	77.6690	-1.2790
63.9920	74.9200	1.5320	63.9920	74.9200	-1.5320
70.9620	72.0710	1.7990	70.9620	72.0710	-1.7990
78.1910	69.1160	1.6170	78.1910	69.1160	-1.6170
85.6920	66.0500	1.4240	85.6920	66.0500	-1.4240
51.0320	65.3390	1.4330	51.0320	65.3390	-1.4330
58.2970	62.3690	1.7150	58.2970	62.3690	-1.7150
65.8260	59.2910	2.0120	65.8260	59.2910	-2.0120
73.6350	56.1000	1.8070	73.6350	56.1000	-1.8070
81.7380	52.7870	1.5900	81.7380	52.7870	-1.5900
44.7990	53.0080	1.5670	44.7990	53.0080	-1.5670
52.6030	49.8180	1.8980	52.6030	49.8180	-1.8980
60.6310	46.5120	2.2250	60.6310	46.5120	-2.2250
69.0790	43.0830	1.9970	69.0790	43.0830	-1.9970
77.7640	39.5250	1.7560	77.7640	39.5250	-1.7560
38.5650	40.6730	1.7420	38.5650	40.6730	-1.7420
46.9080	37.2670	2.0820	46.9080	37.2670	-2.0820

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 69.8760   13.0000     2.0580   69.8760    13.0000   -2.0580
 25.1560   14.1730     2.0730   25.1560    14.1730   -2.0730
 35.5830   12.3040     2.4460   35.5830    12.3040   -2.4460
 46.1310   10.4030     2.8270   46.1310    10.4030   -2.8270
 56.3640    8.4690     2.5020   56.3640     8.4690   -2.5020
 67.9380    6.5000     2.1690   67.9380     6.5000   -2.1690
 18.0000    0.0000     2.2500   18.0000     0.0000   -2.2500
 30.0000    0.0000     2.6250   30.0000     0.0000   -2.6250
 42.0000    0.0000     3.0000   42.0000     0.0000   -3.0000
 54.0000    0.0000     2.6250   54.0000     0.0000   -2.6250
 66.0000    0.0000     2.2500   66.0000     0.0000   -2.2500
235  236  237  238  239  240  241  242  243  244  245  246  247  248
249  250  251  252  253  254  255  256  257  258  259  260  261  262
263  264
142  142
 .290E+02   3   3   .290E+02   3    4  -.280E+04   1    5
-.596E+04   2   5   .113E+04   3    5   .260E+04   1    6
 .596E+04   2   6   .113E+04   3    6   .309E+02   3    7
 .309E+02   3   8  -.987E+04   1    9  -.978E+04   2    9
 .113E+04   3   9   .987E+04   1   10   .978E+04   2   10
 .113E+04   3  10   .205E+03   1    1  -.738E+04   2    1
 .926E+03   3   1  -.205E+03   1    2   .738E+04   2    2
 .926E+03   3   2   .178E+03   3   11   .179E+03   3   12
 .214E+03   3  13   .214E+03   3   14   .253E+03   3   15
 .253E+03   3  15  -.568E+04   1   17   .232E+04   2   17
 .102E+04   3  17   .568E+04   1   18  -.232E+04   2   18
 .102E+04   3  18   .231E+04   1   19  -.946E+03   2   19
 .723E+03   3  19  -.231E+04   1   20   .946E+03   2   20
 .723E+03   3  20   .314E+03   3   21   .314E+03   3   22
 .326E+03   3  23   .326E+03   3   24   .338E+03   3   25
 .338E+03   3  25  -.407E+04   1   27   .166E+04   2   27
 .902E+03   3  27   .407E+04   1   28  -.166E+04   2   28
 .902E+03   3  28   .174E+04   1   29  -.713E+03   2   29
 .646E+03   3  29  -.174E+04   1   30   .713E+03   2   30
 .646E+03   3  30   .340E+03   3   31   .340E+03   3   32
 .352E+03   3  33   .352E+03   3   34   .365E+03   3   35
 .365E+03   3  36  -.425E+04   1   37   .174E+04   2   37
 .974E+03   3  37   .425E+04   1   38  -.174E+04   2   38
 .974E+03   3  38   .182E+04   1   39  -.743E+03   2   39
 .694E+03   3  39  -.182E+04   1   40   .743E+03   2   40
 .694E+03   3  40   .365E+03   3   41   .365E+03   3   42
 .378E+03   3  43   .378E+03   3   44   .392E+03   3   45
 .392E+03   3  45  -.444E+04   1   47   .182E+04   2   47
 .105E+04   3  47   .444E+04   1   48  -.182E+04   2   48
 .105E+04   3  48   .189E+04   1   49  -.773E+03   2   49
 .742E+03   3  49  -.189E+04   1   50   .773E+03   2   50
 .742E+03   3  50   .390E+03   3   51   .390E+03   3   52
 .404E+03   3  53   .404E+03   3   54   .420E+03   3   55
 .420E+03   3  56  -.464E+04   1   57   .190E+04   2   57
 .112E+04   3  57   .464E+04   1   58  -.190E+04   2   58
 .112E+04   3  58   .229E+04   1   59  -.937E+03   2   59
 .883E+03   3  59  -.229E+04   1   60   .937E+03   2   60
 .883E+03   3  60   .413E+03   3   61   .413E+03   3   62
 .391E+03   3  63   .391E+03   3   64   .368E+03   3   65
 .368E+03   3  66  -.303E+04   1   67   .124E+04   2   67
 .804E+03   3  67   .303E+04   1   68  -.124E+04   2   68
 .804E+03   3  68   .307E+04   1   69  -.520E+03   2   69
 .104E+04   3  69  -.307E+04   1   70   .520E+03   2   70
 .104E+04   3  70   .433E+03   3   71   .433E+03   3   72
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.370E+03	3	73	.370E+03	3	74	.304E+03	3	75
.304E+03	3	76	-.137E+04	1	77	.262E+03	2	77
.446E+03	3	77	.137E+04	1	78	.262E+03	2	78
.446E+03	3	78						
.295E+02	3	3	.295E+02	3	4	-.242E+04	1	5
-.602E+04	2	5	.979E+03	3	5	.242E+04	1	6
.602E+04	2	6	.979E+03	3	6	.553E+02	3	7
.559E+02	3	8	-.402E+04	1	9	-.398E+04	2	9
.474E+03	3	9	.402E+04	1	10	.398E+04	2	10
.474E+03	3	10	.351E+03	1	1	-.126E+05	2	1
.153E+04	3	1	-.351E+03	1	2	.126E+05	2	2
.153E+04	3	2	.194E+03	3	11	.194E+03	3	12
.175E+03	3	13	.175E+03	3	14	.157E+03	3	15
.157E+03	3	16	-.160E+04	1	17	.653E+03	2	17
.325E+03	3	17	.160E+04	1	18	-.653E+03	2	18
.325E+03	3	18	.551E+04	1	19	-.225E+04	2	19
.155E+04	3	19	-.551E+04	1	20	.225E+04	2	20
.155E+04	3	20	.347E+03	3	21	.347E+03	3	22
.270E+03	3	23	.270E+03	3	24	.213E+03	3	25
.213E+03	3	26	-.121E+04	1	27	.496E+03	2	27
.311E+03	3	27	.121E+04	1	28	-.496E+03	2	28
.311E+03	3	28	.399E+04	1	29	-.163E+04	2	29
.131E+04	3	29	-.399E+04	1	30	.163E+04	2	30
.131E+04	3	30	.375E+03	3	31	.375E+03	3	32
.291E+03	3	33	.291E+03	3	34	.230E+03	3	35
.230E+03	3	36	-.127E+04	1	37	.518E+03	2	37
.336E+03	3	37	.127E+04	1	38	-.518E+03	2	38
.336E+03	3	38	.416E+04	1	39	-.170E+04	2	39
.141E+04	3	39	-.416E+04	1	40	.170E+04	2	40
.141E+04	3	40	.402E+03	3	41	.402E+03	3	42
.313E+03	3	43	.313E+03	3	44	.247E+03	3	45
.247E+03	3	46	-.132E+04	1	47	.541E+03	2	47
.361E+03	3	47	.132E+04	1	48	-.541E+03	2	48
.361E+03	3	48	.433E+04	1	49	-.177E+04	2	49
.150E+04	3	49	-.433E+04	1	50	.177E+04	2	50
.150E+04	3	50	.430E+03	3	51	.430E+03	3	52
.334E+03	3	53	.334E+03	3	54	.264E+03	3	55
.264E+03	3	56	-.138E+04	1	57	.565E+03	2	57
.386E+03	3	57	.138E+04	1	58	-.565E+03	2	58
.386E+03	3	58	.530E+04	1	59	-.217E+04	2	59
.182E+04	3	59	-.530E+04	1	60	.217E+04	2	60
.182E+04	3	60	.458E+03	3	61	.458E+03	3	62
.326E+03	3	63	.326E+03	3	64	.233E+03	3	65
.233E+03	3	66	-.922E+03	1	67	.377E+03	2	67
.287E+03	3	67	.922E+03	1	68	-.377E+03	2	68
.287E+03	3	68	.716E+04	1	69	-.121E+04	2	69
.218E+04	3	69	-.716E+04	1	70	.121E+04	2	70
.218E+04	3	70	.484E+03	3	71	.484E+03	3	72
.310E+03	3	73	.310E+03	3	74	.194E+03	3	75
.194E+03	3	76	-.451E+03	1	77	.360E+02	2	77
.175E+03	3	77	.451E+03	1	78	-.860E+02	2	78
.175E+03	3	78						

APPENDIX D : RESULTS OF THE SAMPLE PROBLEM ANALYSIS

PAGE 1

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
1	.031	9.28	32	1	3	11		.1150E+05	-.1072E+05	-.7223E+04	.2296E+05				.6297E+01	.162E+01
								.1630E+05	-.1321E+05	-.1825E+05	.4067E+05				.1978E+02	.484E+00
2	.031	9.28	32	2	4	12		-.1150E+05	.1072E+05	.7223E+04	.2296E+05				.6297E+01	.162E+01
								-.1630E+05	.1321E+05	.1825E+05	.4067E+05				.1978E+02	.484E+00
3	.031	28.51	42	3	5	13	11	-.2799E+05	-.2990E+05	.3225E+05	.8821E+00	.6294E+05	.8299E+00	.7833E+00	.1295E+03	.769E-01
								-.1677E+05	-.2533E+05	.1865E+05	.8356E+00	.3926E+05	.7585E+00	.8513E+00	.5026E+02	.760E+00
4	.031	28.51	42	4	6	14	12	.2799E+05	.2990E+05	-.3225E+05	.8821E+00	.6294E+05	.8299E+00	.7833E+00	.1295E+03	.769E-01
								.1677E+05	.2533E+05	-.1865E+05	.8356E+00	.3926E+05	.7585E+00	.8513E+00	.5026E+02	.760E+00
5	.031	48.79	42	5	7	15	13	-.1431E+05	-.4576E+05	.1269E+05	.6114E+00	.4567E+00	.7033E+00	.4612E+05	.8417E+02	.959E+00
								-.8232E+02	-.3900E+05	.1240E+04	.7730E+00	.5601E+00	.6495E+00	.3902E+05	.5953E+02	.113E+01
6	.031	48.79	42	6	8	16	14	.1431E+05	.4576E+05	-.1269E+05	.6114E+00	.4567E+00	.7033E+00	.4612E+05	.8417E+02	.959E+00
								.8232E+02	.3900E+05	-.1240E+04	.7730E+00	.5601E+00	.6495E+00	.3902E+05	.5953E+02	.113E+01
7	.042	70.24	42	7	9	17	15	-.3013E+05	-.4199E+05	.3984E+05	.9941E+00	.7853E+05	.6949E+00	.6979E+00	.6326E+03	-.100E+00
								-.1027E+05	-.2025E+05	.1844E+05	.9579E+00	.3643E+05	.6815E+00	.6397E+00	.1271E+03	.998E+00
8	.042	70.24	42	8	10	18	16	.3013E+05	.4199E+05	-.3984E+05	.9941E+00	.7853E+05	.6949E+00	.6979E+00	.6326E+03	-.100E+00
								.1027E+05	.2025E+05	-.1844E+05	.9579E+00	.3643E+05	.6815E+00	.6397E+00	.1271E+03	.998E+00
9	.031	96.12	42	1	11	21	19	-.5404E+04	-.5376E+05	-.1147E+05	.9576E+00	.8637E+00	.9235E+00	.5499E+05	.3905E+03	.168E+00
								-.1695E+05	-.5036E+05	-.3445E+05	.7437E+05	.7840E+00	.7550E+00	.9578E+00	.6031E+03	-.695E-01
10	.031	96.12	42	2	12	22	20	.5404E+04	.5376E+05	.1147E+05	.9576E+00	.8637E+00	.9235E+00	.5499E+05	.3905E+03	.168E+00
								.1695E+05	.5036E+05	.3445E+05	.7437E+05	.7840E+00	.7550E+00	.9578E+00	.6031E+03	-.695E-01
11	.031	99.62	42	11	13	23	21	-.6569E+04	-.6259E+05	.1393E+05	.9258E+00	.6427E+05	.8560E+00	.7520E+00	.4990E+03	.596E-01
								-.9696E+04	-.4683E+05	-.1269E+04	.8641E+00	.4287E+05	.9388E+00	.7713E+00	.2473E+03	.565E+00
12	.031	99.62	42	12	14	24	22	.6569E+04	.6259E+05	-.1393E+05	.9258E+00	.6427E+05	.8560E+00	.7520E+00	.4990E+03	.596E-01
								.9696E+04	.4683E+05	.1269E+04	.8641E+00	.4287E+05	.9388E+00	.7713E+00	.2473E+03	.565E+00
13	.036	103.28	42	13	15	25	23	-.2933E+04	-.6248E+05	.1378E+03	.9226E+00	.9765E+00	.6107E+05	.9327E+00	.6557E+03	.248E-01
								-.4382E+04	-.4428E+05	-.1371E+05	.9538E+00	.9712E+00	.4847E+05	.9755E+00	.4125E+03	.272E+00
14	.036	103.28	42	14	16	26	24	.2933E+04	.6248E+05	-.1378E+03	.9226E+00	.9765E+00	.6107E+05	.9327E+00	.6557E+03	.248E-01
								.4382E+04	.4428E+05	.1371E+05	.9538E+00	.9712E+00	.4847E+05	.9755E+00	.4125E+03	.272E+00
15	.036	107.18	42	15	17	27	25	.4674E+02	-.5888E+05	.1810E+05	.8819E+00	.8912E+00	.6673E+05	.9853E+00	.7416E+03	-.422E-01
								-.6999E+02	-.4121E+05	.3025E+04	.9163E+00	.9131E+00	.4151E+05	.9963E+00	.3041E+03	.510E+00
16	.036	107.18	42	16	18	28	26	-.4674E+02	.5888E+05	-.1810E+05	.8819E+00	.8912E+00	.6673E+05	.9853E+00	.7416E+03	-.422E-01
								.6999E+02	.4121E+05	-.3025E+04	.9163E+00	.9131E+00	.4151E+05	.9963E+00	.3041E+03	.510E+00
17	.036	104.16	42	19	21	31	29	-.1214E+04	-.6501E+05	-.7260E+04	.8748E+00	.6562E+05	.9551E+00	.8262E+00	.6621E+03	-.306E-05
								-.1743E+04	-.4980E+05	-.2551E+05	.8177E+00	.9962E+00	.6595E+05	.8302E+00	.6464E+03	.746E-03

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
18	.036	104.16	42	20	22	32	30	.1214E+04	.6501E+05	.7260E+04	.8748E+00	.6562E+05	.9551E+00	.8262E+00	.6621E+03	-.306E-05
								.1743E+04	.4980E+05	.2551E+05	.8177E+00	.9962E+00	.6595E+05	.8302E+00	.6464E+03	.746E-03
19	.047	107.95	42	21	23	33	31	-.3887E+04	-.7808E+05	.8556E+04	.9190E+00	.7764E+05	.9303E+00	.8400E+00	.1271E+04	-.162E+00
								-.7068E+04	-.6300E+05	-.4625E+04	.9217E+00	.6031E+05	.9904E+00	.9062E+00	.8438E+03	.417E-01
20	.047	107.95	42	22	24	34	32	.3887E+04	.7808E+05	-.8556E+04	.9190E+00	.7764E+05	.9303E+00	.8400E+00	.1271E+04	-.162E+00
								.7068E+04	.6300E+05	.4625E+04	.9217E+00	.6031E+05	.9904E+00	.9062E+00	.8438E+03	.417E-01
21	.047	111.92	42	23	25	35	33	-.3478E+04	-.7611E+05	-.3260E+04	.9545E+00	.9289E+00	.9762E+00	.7464E+05	.1325E+04	-.167E+00
								-.4866E+04	-.6073E+05	-.1670E+05	.9668E+00	.9221E+00	.9593E+00	.6522E+05	.9878E+03	-.416E-01
22	.047	111.92	42	24	26	36	34	.3478E+04	.7611E+05	.3260E+04	.9545E+00	.9289E+00	.9762E+00	.7464E+05	.1325E+04	-.167E+00
								.4866E+04	.6073E+05	.1670E+05	.9668E+00	.9221E+00	.9593E+00	.6522E+05	.9878E+03	-.416E-01
23	.052	116.13	42	25	27	37	35	-.2952E+03	-.7057E+05	.1344E+05	.9426E+00	.9272E+00	.9834E+00	.7417E+05	.1457E+04	-.160E+00
								-.1711E+03	-.5466E+05	-.6348E+03	.9340E+00	.8934E+00	.9610E+00	.5459E+05	.7758E+03	.160E+00
24	.052	116.13	42	26	28	38	36	.2952E+03	.7057E+05	-.1344E+05	.9426E+00	.9272E+00	.9834E+00	.7417E+05	.1457E+04	-.160E+00
								.1711E+03	.5466E+05	.6348E+03	.9340E+00	.8934E+00	.9610E+00	.5459E+05	.7758E+03	.160E+00
25	.042	112.19	42	29	31	41	39	-.8957E+03	-.7312E+05	-.7545E+04	.9307E+00	.7384E+05	.9628E+00	.8933E+00	.1096E+04	-.142E+00
								-.3554E+04	-.6398E+05	-.2319E+05	.9448E+00	.7411E+05	.9960E+00	.9430E+00	.1128E+04	-.164E+00
26	.042	112.19	42	30	32	42	40	.8957E+03	.7312E+05	.7545E+04	.9307E+00	.7384E+05	.9628E+00	.8933E+00	.1096E+04	-.142E+00
								.3554E+04	.6398E+05	.2319E+05	.9448E+00	.7411E+05	.9960E+00	.9430E+00	.1128E+04	-.164E+00
27	.062	116.27	42	31	33	43	41	-.1184E+04	-.8523E+05	.6317E+04	.9283E+00	.8535E+05	.9280E+00	.8518E+00	.2196E+04	-.241E+00
								-.3919E+04	-.7137E+05	-.5073E+04	.9270E+00	.7005E+05	.9595E+00	.8850E+00	.1554E+04	-.916E-01
28	.062	116.27	42	32	34	44	42	.1184E+04	.8523E+05	-.6317E+04	.9283E+00	.8535E+05	.9280E+00	.8518E+00	.2196E+04	-.241E+00
								.3919E+04	.7137E+05	.5073E+04	.9270E+00	.7005E+05	.9595E+00	.8850E+00	.1554E+04	-.916E-01
29	.062	120.54	42	33	35	45	43	-.2180E+04	-.8410E+05	-.6116E+04	.9700E+00	.9637E+00	.9947E+00	.8370E+05	.2448E+04	-.270E+00
								-.3938E+04	-.7010E+05	-.1898E+05	.9763E+00	.9541E+00	.9796E+00	.7572E+05	.1955E+04	-.188E+00
30	.062	120.54	42	34	36	46	44	.2180E+04	.8410E+05	.6116E+04	.9700E+00	.9637E+00	.9947E+00	.8370E+05	.2448E+04	-.270E+00
								.3938E+04	.7010E+05	.1898E+05	.9763E+00	.9541E+00	.9796E+00	.7572E+05	.1955E+04	-.188E+00
31	.062	125.10	42	35	37	47	45	-.1330E+03	-.8168E+05	.1163E+05	.9486E+00	.9356E+00	.9866E+00	.8407E+05	.2452E+04	-.262E+00
								-.2666E+03	-.6656E+05	-.2410E+04	.9436E+00	.9048E+00	.9623E+00	.6655E+05	.1506E+04	-.538E-01
32	.062	125.10	42	36	38	48	46	.1330E+03	.8168E+05	-.1163E+05	.9486E+00	.9356E+00	.9866E+00	.8407E+05	.2452E+04	-.262E+00
								.2666E+03	.6656E+05	.2410E+04	.9436E+00	.9048E+00	.9623E+00	.6655E+05	.1506E+04	-.538E-01
33	.047	120.22	42	39	41	51	49	-.2290E+04	-.8144E+05	-.6196E+04	.9351E+00	.8103E+05	.9466E+00	.8813E+00	.1586E+04	-.213E+00
								-.4952E+04	-.7279E+05	-.2167E+05	.9517E+00	.7982E+05	.9827E+00	.9364E+00	.1593E+04	-.221E+00
34	.047	120.22	42	40	42	52	50	.2290E+04	.8144E+05	.6196E+04	.9351E+00	.8103E+05	.9466E+00	.8813E+00	.1586E+04	-.213E+00
								.4952E+04	.7279E+05	.2167E+05	.9517E+00	.7982E+05	.9827E+00	.9364E+00	.1593E+04	-.221E+00

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
35	.068	124.61	42	41	43	53	51	-.2535E+04	-.9870E+05	.3706E+04	.9144E+00	.9767E+05	.9045E+00	.8143E+00	.3242E+04	-.324E+00
								-.5892E+04	-.8515E+05	-.6821E+04	.9146E+00	.8321E+05	.9307E+00	.8438E+00	.2461E+04	-.218E+00
36	.068	124.61	42	42	44	54	52	.2535E+04	.9870E+05	-.3706E+04	.9144E+00	.9767E+05	.9045E+00	.8143E+00	.3242E+04	-.324E+00
								.5892E+04	.8515E+05	.6821E+04	.9146E+00	.8321E+05	.9307E+00	.8438E+00	.2461E+04	-.218E+00
37	.073	129.17	42	43	45	55	53	-.2514E+04	-.9764E+05	-.1000E+05	.9707E+00	.9409E+00	.9710E+00	.9795E+05	.4088E+04	-.369E+00
								-.5039E+04	-.8392E+05	-.2218E+05	.9770E+00	.9344E+00	.9588E+00	.9012E+05	.3404E+04	-.310E+00
38	.073	129.17	42	44	46	56	54	.2514E+04	.9764E+05	.1000E+05	.9707E+00	.9409E+00	.9710E+00	.9795E+05	.4088E+04	-.369E+00
								.5039E+04	.8392E+05	.2218E+05	.9770E+00	.9344E+00	.9588E+00	.9012E+05	.3404E+04	-.310E+00
39	.078	134.06	42	45	47	57	55	.6827E+03	-.9273E+05	.8767E+04	.9358E+00	.8944E+00	.9581E+00	.9430E+05	.3974E+04	-.328E+00
								-.8048E+02	-.7785E+05	-.4519E+04	.9302E+00	.8580E+00	.9295E+00	.7821E+05	.2656E+04	-.174E+00
40	.078	134.06	42	46	48	58	56	-.6827E+03	.9273E+05	-.8767E+04	.9358E+00	.8944E+00	.9581E+00	.9430E+05	.3974E+04	-.328E+00
								.8048E+02	.7785E+05	.4519E+04	.9302E+00	.8580E+00	.9295E+00	.7821E+05	.2656E+04	-.174E+00
41	.052	128.26	42	49	51	61	59	-.4164E+04	-.8117E+05	-.4440E+04	.9479E+00	.7955E+05	.9389E+00	.8863E+00	.1841E+04	-.200E+00
								-.7082E+04	-.7521E+05	-.1894E+05	.9589E+00	.7906E+05	.9714E+00	.9316E+00	.1879E+04	-.213E+00
42	.052	128.26	42	50	52	62	60	.4164E+04	.8117E+05	.4440E+04	.9479E+00	.7955E+05	.9389E+00	.8863E+00	.1841E+04	-.200E+00
								.7082E+04	.7521E+05	.1894E+05	.9589E+00	.7906E+05	.9714E+00	.9316E+00	.1879E+04	-.213E+00
43	.073	132.92	42	51	53	63	61	-.5741E+04	-.1002E+06	-.2650E+03	.9186E+00	.9749E+05	.8897E+00	.8046E+00	.3730E+04	-.318E+00
								-.1017E+05	-.8885E+05	-.8870E+04	.9277E+00	.8562E+05	.9199E+00	.8464E+00	.3058E+04	-.241E+00
44	.073	132.92	42	52	54	64	62	.5741E+04	.1002E+06	.2650E+03	.9186E+00	.9749E+05	.8897E+00	.8046E+00	.3730E+04	-.318E+00
								.1017E+05	.8885E+05	.8870E+04	.9277E+00	.8562E+05	.9199E+00	.8464E+00	.3058E+04	-.241E+00
45	.094	137.80	42	53	55	65	63	-.4183E+04	-.9895E+05	-.1461E+05	.9880E+00	.9451E+00	.9577E+00	.1002E+06	.5896E+04	-.384E+00
								-.7067E+04	-.8715E+05	-.2491E+05	.9869E+00	.9307E+00	.9448E+00	.9430E+05	.5106E+04	-.339E+00
46	.094	137.80	42	54	56	66	64	.4183E+04	.9895E+05	.1461E+05	.9880E+00	.9451E+00	.9577E+00	.1002E+06	.5896E+04	-.384E+00
								.7067E+04	.8715E+05	.2491E+05	.9869E+00	.9307E+00	.9448E+00	.9430E+05	.5106E+04	-.339E+00
47	.104	143.02	42	55	57	67	65	.3288E+03	-.9420E+05	.4538E+04	.9577E+00	.9176E+00	.9596E+00	.9469E+05	.5853E+04	-.339E+00
								-.7511E+03	-.8004E+05	-.6725E+04	.9485E+00	.8774E+00	.9299E+00	.8051E+05	.4085E+04	-.206E+00
48	.104	143.02	42	56	58	68	66	-.3288E+03	.9420E+05	-.4538E+04	.9577E+00	.9176E+00	.9596E+00	.9469E+05	.5853E+04	-.339E+00
								.7511E+03	.8004E+05	.6725E+04	.9485E+00	.8774E+00	.9299E+00	.8051E+05	.4085E+04	-.206E+00
49	.052	146.00	42	59	61	71	69	-.1008E+05	-.7653E+05	-.2280E+04	.9555E+00	.7213E+05	.8764E+00	.8378E+00	.1698E+04	-.897E-01
								-.1388E+05	-.7498E+05	-.1692E+05	.9427E+00	.7505E+05	.9165E+00	.8708E+00	.1881E+04	-.139E+00
50	.052	146.00	42	60	62	72	70	.1008E+05	.7653E+05	.2280E+04	.9555E+00	.7213E+05	.8764E+00	.8378E+00	.1698E+04	-.897E-01
								.1388E+05	.7498E+05	.1692E+05	.9427E+00	.7505E+05	.9165E+00	.8708E+00	.1881E+04	-.139E+00
51	.078	127.77	42	61	63	73	71	-.1190E+05	-.9611E+05	-.3685E+04	.9116E+00	.9098E+05	.8643E+00	.7920E+00	.3337E+04	-.256E+00
								-.1700E+05	-.8836E+05	-.1057E+05	.9302E+00	.8324E+05	.9025E+00	.8479E+00	.3039E+04	-.213E+00

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
52	.078	127.77	42	62	64	74	72	.1190E+05	.9611E+05	.3685E+04	.9116E+00	.9098E+05	.8643E+00	.7920E+00	.3337E+04	-.256E+00
								.1700E+05	.8836E+05	.1057E+05	.9302E+00	.8324E+05	.9025E+00	.8479E+00	.3039E+04	-.213E+00
53	.114	107.85	42	63	65	75	73	-.6871E+04	-.9669E+05	-.1986E+05	.9958E+05	.9768E+00	.9366E+00	.9623E+00	.5646E+04	-.377E+00
								-.1044E+05	-.8613E+05	-.2869E+05	.9538E+05	.9600E+00	.9414E+00	.9814E+00	.5158E+04	-.350E+00
54	.114	107.85	42	64	66	76	74	.6871E+04	.9669E+05	.1986E+05	.9958E+05	.9768E+00	.9366E+00	.9623E+00	.5646E+04	-.377E+00
								.1044E+05	.8613E+05	.2869E+05	.9538E+05	.9600E+00	.9414E+00	.9814E+00	.5158E+04	-.350E+00
55	.120	86.21	42	65	67	77	75	-.5142E+04	-.1055E+06	-.5026E+04	.9540E+00	.1034E+06	.9344E+00	.9009E+00	.4783E+04	-.385E+00
								-.4711E+04	-.8353E+05	-.1274E+05	.8422E+05	.9994E+00	.9671E+00	.9718E+00	.3447E+04	-.275E+00
56	.120	86.21	42	66	68	78	76	.5142E+04	.1055E+06	.5026E+04	.9540E+00	.1034E+06	.9344E+00	.9009E+00	.4783E+04	-.385E+00
								.4711E+04	.8353E+05	.1274E+05	.8422E+05	.9994E+00	.9671E+00	.9718E+00	.3447E+04	-.275E+00
57	.047	154.48	42	69	71	81	79	-.8991E+04	-.5774E+05	.1264E+05	.8564E+00	.5809E+05	.7597E+00	.6361E+00	.8280E+03	.284E+00
								-.1319E+05	-.6258E+05	.1173E+04	.9624E+00	.5717E+05	.9229E+00	.8978E+00	.1119E+04	.112E+00
58	.047	154.48	42	70	72	82	80	.8991E+04	.5774E+05	-.1264E+05	.8564E+00	.5809E+05	.7597E+00	.6361E+00	.8280E+03	.284E+00
								.1319E+05	.6258E+05	-.1173E+04	.9624E+00	.5717E+05	.9229E+00	.8978E+00	.1119E+04	.112E+00
59	.088	133.04	42	71	73	83	81	-.1496E+05	-.9825E+05	.1254E+05	.8348E+00	.9423E+05	.7451E+00	.6164E+00	.3488E+04	-.194E+00
								-.1792E+05	-.9922E+05	.5423E+04	.8238E+00	.9207E+05	.7552E+00	.6233E+00	.3432E+04	-.177E+00
60	.088	133.04	42	72	74	84	82	.1496E+05	.9825E+05	-.1254E+05	.8348E+00	.9423E+05	.7451E+00	.6164E+00	.3488E+04	-.194E+00
								.1792E+05	.9922E+05	-.5423E+04	.8238E+00	.9207E+05	.7552E+00	.6233E+00	.3432E+04	-.177E+00
61	.125	111.00	42	73	75	85	83	-.1676E+05	-.1030E+06	.4519E+04	.9402E+00	.8096E+00	.8914E+00	.9606E+05	.5562E+04	-.317E+00
								-.1743E+05	-.1037E+06	-.3232E+04	.9211E+00	.7478E+00	.8545E+00	.9638E+05	.5317E+04	-.298E+00
62	.125	111.00	42	74	76	86	84	.1676E+05	.1030E+06	-.4519E+04	.9402E+00	.8096E+00	.8914E+00	.9606E+05	.5562E+04	-.317E+00
								.1743E+05	.1037E+06	.3232E+04	.9211E+00	.7478E+00	.8545E+00	.9638E+05	.5317E+04	-.298E+00
63	.140	88.43	42	75	77	87	85	-.1974E+05	-.1330E+06	-.9329E+03	.8307E+00	.1243E+06	.7206E+00	.6100E+00	.6185E+04	-.378E+00
								-.1822E+05	-.1085E+06	-.1128E+05	.8707E+00	.1025E+06	.7400E+00	.6621E+00	.4523E+04	-.274E+00
64	.140	88.43	42	76	78	88	86	.1974E+05	.1330E+06	.9329E+03	.8307E+00	.1243E+06	.7206E+00	.6100E+00	.6185E+04	-.378E+00
								.1822E+05	.1085E+06	.1128E+05	.8707E+00	.1025E+06	.7400E+00	.6621E+00	.4523E+04	-.274E+00
65	.019	17.88	51	1	2	4	3			-.9111E+04	.7004E+00	.1000E+01	.7433E+00	.9111E+04	.2660E+01	.346E+01
										.1206E+05	.1206E+05	.6665E+00	.9522E+00	.6665E+00	.4266E+01	.254E+01
66	.019	20.63	51	3	4	6	5			-.6863E+04	.6863E+04	.4466E+00	.9310E+00	.4466E+00	.1290E+01	.624E+01
										.9370E+04	.7972E+00	.1000E+01	.8225E+00	.9370E+04	.3530E+01	.313E+01
67	.019	20.63	51	5	6	8	7			-.1524E+05	.9795E+00	.1000E+01	.9766E+00	.1524E+05	.1103E+02	.132E+01
										-.2907E+04	.9249E+00	.3980E+00	.2907E+04	.3980E+00	.2217E+00	.167E+02
68	.019	17.88	51	7	8	10	9			-.1918E+05	.6865E+00	.1918E+05	.6342E+00	.1000E+01	.1113E+02	.120E+01
										.1267E+04	.7208E+00	.9500E+00	.1267E+04	.9500E+00	.5547E-01	.297E+02

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
69	.019	16.53	51	1	2	12	11			.1604E+05	.1604E+05	.9315E+00	.9886E+00	.9315E+00	.9283E+01	.127E+01
										.1542E+05	.9933E+00	.1000E+01	.9944E+00	.1542E+05	.9190E+01	.128E+01
70	.019	20.33	51	11	12	14	13			.1711E+05	.1711E+05	.9722E+00	.9958E+00	.9722E+00	.1358E+02	.108E+01
										.1741E+05	.9738E+00	.1000E+01	.9777E+00	.1741E+05	.1415E+02	.103E+01
71	.019	21.64	51	13	14	16	15			.6507E+04	.9694E+00	.1000E+01	.9660E+00	.6507E+04	.2087E+01	.447E+01
										.6298E+03	.8215E+00	.1000E+01	.8016E+00	.6298E+03	.1676E-01	.603E+02
72	.019	20.02	51	15	16	18	17			.1861E+05	.9780E+00	.1000E+01	.9750E+00	.1861E+05	.1594E+02	.903E+00
										.7627E+04	.9238E+00	.1000E+01	.9137E+00	.7627E+04	.2528E+01	.378E+01
73	.019	20.42	51	19	20	22	21			.5618E+04	.5618E+04	.5699E+00	.9290E+00	.5699E+00	.9479E+00	.714E+01
										.2101E+04	.2101E+04	.5648E+00	.9281E+00	.5648E+00	.1319E+00	.208E+02
74	.019	25.08	51	21	22	24	23			.1324E+05	.1324E+05	.8481E+00	.9775E+00	.8481E+00	.8768E+01	.188E+01
										.1142E+05	.1142E+05	.9108E+00	.9868E+00	.9108E+00	.6979E+01	.222E+01
75	.019	26.68	51	23	24	26	25			-.1243E+05	.9867E+00	.8690E+00	.1243E+05	.8690E+00	.8448E+01	.202E+01
										-.8234E+04	.9877E+00	.8784E+00	.8234E+04	.8784E+00	.3741E+01	.354E+01
76	.019	24.64	51	25	26	28	27			-.3760E+04	.9210E+00	.3378E+00	.3760E+04	.3378E+00	.4234E+00	.134E+02
										-.2709E+04	.9186E+00	.3182E+00	.2709E+04	.3182E+00	.2165E+00	.193E+02
77	.019	24.71	51	29	30	32	31			.9061E+04	.9061E+04	.6170E+00	.9370E+00	.6170E+00	.3136E+01	.388E+01
										.1785E+05	.1785E+05	.8510E+00	.9755E+00	.8510E+00	.1572E+02	.113E+01
78	.019	30.31	51	31	32	34	33			.1558E+05	.1558E+05	.8066E+00	.9715E+00	.8066E+00	.1403E+02	.151E+01
										.2239E+05	.2239E+05	.8983E+00	.9850E+00	.8983E+00	.3202E+02	.654E+00
79	.019	32.22	51	33	34	36	35			-.7198E+04	.9724E+00	.7288E+00	.7198E+04	.7288E+00	.2950E+01	.467E+01
										-.4738E+04	.9697E+00	.7030E+00	.4738E+04	.7030E+00	.1244E+01	.776E+01
80	.019	29.74	51	35	36	38	37			.5312E+04	.5420E+00	.1000E+01	.4795E+00	.5312E+04	.1248E+01	.771E+01
										.3777E+04	.4824E+00	.1000E+01	.4118E+00	.3777E+04	.6005E+00	.118E+02
81	.019	29.38	51	39	40	42	41			.1107E+05	.1107E+05	.6622E+00	.9446E+00	.6622E+00	.5839E+01	.288E+01
										.1803E+05	.1803E+05	.8320E+00	.9725E+00	.8320E+00	.1869E+02	.114E+01
82	.019	36.03	51	41	42	44	43			.1677E+05	.1677E+05	.8029E+00	.9710E+00	.8029E+00	.1923E+02	.134E+01
										.2222E+05	.2222E+05	.8851E+00	.9831E+00	.8851E+00	.3694E+02	.679E+00
83	.019	38.26	51	43	44	46	45			-.2998E+04	.9208E+00	.2275E+00	.2998E+04	.2275E+00	.3929E+00	.187E+02
										-.3271E+04	.9463E+00	.4761E+00	.3271E+04	.4761E+00	.5642E+00	.138E+02
84	.019	35.29	51	45	46	48	47			.1080E+05	.7385E+00	.1080E+05	.7026E+00	.1000E+01	.7365E+01	.276E+01
										.5858E+04	.6141E+00	.1000E+01	.5612E+00	.5858E+04	.1920E+01	.652E+01
85	.019	34.47	51	49	50	52	51			.2118E+05	.2118E+05	.8250E+00	.9714E+00	.8250E+00	.3000E+02	.827E+00
										.2915E+05	.2915E+05	.8932E+00	.9826E+00	.8932E+00	.6129E+02	.275E+00

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
86	.019	42.22	51	51	52	54	53			.2382E+05	.2382E+05	.8621E+00	.9799E+00	.8621E+00	.4852E+02	.587E+00
										.2996E+05	.2996E+05	.9167E+00	.9878E+00	.9167E+00	.8146E+02	.223E+00
87	.019	44.81	51	53	54	56	55			.5239E+04	.5939E+00	.5239E+04	.5473E+00	.1000E+01	.1920E+01	.750E+01
										.1077E+04	.4567E+00	.1000E+01	.6239E+00	.1077E+04	.7915E-01	.413E+02
88	.019	41.32	51	55	56	58	57			.1824E+05	.8404E+00	.1000E+01	.8184E+00	.1824E+05	.2730E+02	.110E+01
										.9185E+04	.7438E+00	.1000E+01	.7085E+00	.9185E+04	.6270E+01	.341E+01
89	.022	39.93	51	59	60	62	61			.3399E+05	.3399E+05	.9016E+00	.9840E+00	.9016E+00	.1128E+03	.880E-01
										.4273E+05	.4273E+05	.9334E+00	.9891E+00	.9334E+00	.1846E+03	-.150E+00
90	.021	48.89	51	61	62	64	63			.3281E+05	.3281E+05	.9212E+00	.9885E+00	.9212E+00	.1256E+03	.114E+00
										.3978E+05	.3978E+05	.9537E+00	.9933E+00	.9537E+00	.1914E+03	-.977E-01
91	.019	51.85	51	63	64	66	65			.2047E+05	.9171E+00	.1000E+01	.9075E+00	.2047E+05	.4682E+02	.788E+00
										.9836E+04	.8774E+00	.1000E+01	.8632E+00	.9836E+04	.1037E+02	.280E+01
92	.019	47.79	51	65	66	68	67			.3553E+05	.9244E+00	.1000E+01	.9140E+00	.3553E+05	.1310E+03	.263E-01
										.1906E+05	.8884E+00	.1000E+01	.8730E+00	.1906E+05	.3627E+02	.952E+00
93	.025	47.83	51	69	70	72	71			.3507E+05	.3507E+05	.9731E+00	.9959E+00	.9731E+00	.1769E+03	.127E-01
										.4035E+05	.4035E+05	.9603E+00	.9939E+00	.9603E+00	.2309E+03	-.114E+00
94	.024	56.78	51	71	72	74	73			.3339E+05	.3339E+05	.9609E+00	.9947E+00	.9609E+00	.1803E+03	.707E-01
										.3765E+05	.3765E+05	.9704E+00	.9960E+00	.9704E+00	.2317E+03	-.555E-01
95	.019	58.38	51	73	74	76	75			.2013E+05	.9299E+00	.1000E+01	.9208E+00	.2013E+05	.5165E+02	.806E+00
										.1044E+05	.8627E+00	.1000E+01	.8448E+00	.1044E+05	.1295E+02	.262E+01
96	.019	52.08	51	75	76	78	77			.3446E+05	.9363E+00	.3446E+05	.9265E+00	.1000E+01	.1359E+03	.514E-01
										.1935E+05	.8909E+00	.1935E+05	.8742E+00	.1000E+01	.4083E+02	.921E+00
97	.019	33.22	51	1	2	20	19			.2823E+05	.5030E+00	.1000E+01	.5628E+00	.2823E+05	.4006E+02	.616E+00
										.2950E+05	.5008E+00	.1000E+01	.5609E+00	.2950E+05	.4368E+02	.548E+00
98	.038	37.47	51	19	20	30	29			.2322E+05	.5048E+00	.1000E+01	.5580E+00	.2322E+05	.6105E+02	.967E+00
										.3619E+05	.7861E+00	.1000E+01	.8091E+00	.3619E+05	.1890E+03	.756E-01
99	.042	41.73	51	29	30	40	39			.2351E+05	.5100E+00	.1000E+01	.5575E+00	.2351E+05	.7717E+02	.940E+00
										.3734E+05	.7469E+00	.1000E+01	.7714E+00	.3734E+05	.2386E+03	.654E-01
100	.046	45.99	51	39	40	50	49			.2147E+05	.5071E+00	.1000E+01	.5510E+00	.2147E+05	.8077E+02	.113E+01
										.3563E+05	.7444E+00	.1000E+01	.7671E+00	.3563E+05	.2727E+03	.119E+00
101	.047	50.27	51	49	50	60	59			.1719E+05	.4849E+00	.1000E+01	.5268E+00	.1719E+05	.5432E+02	.170E+01
										.3306E+05	.7613E+00	.1000E+01	.7807E+00	.3306E+05	.2550E+03	.195E+00
102	.039	63.04	51	59	60	70	69			.7158E+04	.2163E+00	.1000E+01	.1125E+00	.7158E+04	.8023E+01	.742E+01
										.2773E+05	.7138E+00	.1000E+01	.7382E+00	.2773E+05	.1789E+03	.462E+00

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MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
103	.031	68.66	51	69	70	80	79			-.1611E+05	.1611E+05	.7141E+00	.9775E+00	.7141E+00	.5087E+02	.155E+01
										.2108E+05	.7273E+00	.1000E+01	.7488E+00	.2108E+05	.9049E+02	.910E+00
104	.028	17.13	51	5	6	14	13			.3274E+05	.9408E+00	.1008E+01	.9441E+00	.3274E+05	.6008E+02	.101E+00
										.2893E+05	.9529E+00	.2893E+05	.9555E+00	.1000E+01	.4746E+02	.238E+00
105	.037	46.62	51	13	14	24	23			.3590E+05	.6549E+00	.1000E+01	.6957E+00	.3590E+05	.2007E+03	.163E+00
										.3404E+05	.7371E+00	.1000E+01	.7682E+00	.3404E+05	.1940E+03	.173E+00
106	.050	52.49	51	23	24	34	33			.3651E+05	.6604E+00	.1000E+01	.6964E+00	.3651E+05	.3165E+03	.142E+00
										.3336E+05	.7059E+00	.1000E+01	.7370E+00	.3336E+05	.2752E+03	.218E+00
107	.058	58.35	51	33	34	44	43			.3644E+05	.6974E+00	.1000E+01	.7264E+00	.3644E+05	.4196E+03	.122E+00
										.3574E+05	.7450E+00	.1000E+01	.7694E+00	.3574E+05	.4214E+03	.114E+00
108	.065	64.23	51	43	44	54	53			.3587E+05	.7076E+00	.1000E+01	.7331E+00	.3587E+05	.5054E+03	.134E+00
										.3734E+05	.7611E+00	.1000E+01	.7820E+00	.3734E+05	.5750E+03	.580E-01
109	.079	70.09	51	53	54	64	63			.3292E+05	.7315E+00	.1000E+01	.7530E+00	.3292E+05	.5763E+03	.220E+00
										.3682E+05	.7913E+00	.1000E+01	.8081E+00	.3682E+05	.7623E+03	.561E-01
110	.101	62.28	51	63	64	74	73			.2801E+05	.8269E+00	.1000E+01	.8377E+00	.2801E+05	.5173E+03	.364E+00
										.3458E+05	.8742E+00	.1000E+01	.8821E+00	.3458E+05	.8248E+03	.778E-01
111	.126	65.33	51	73	74	84	83			.2707E+05	.8415E+00	.2707E+05	.8507E+00	.1000E+01	.6409E+03	.401E+00
										.3443E+05	.8804E+00	.3443E+05	.8873E+00	.1000E+01	.1076E+04	.790E-01
112	.019	26.59	51	9	10	18	17			.2408E+05	.4719E+00	.1000E+01	.5281E+00	.2408E+05	.2270E+02	.936[illegible]+00
										.1754E+05	.6494E+00	.1000E+01	.6867E+00	.1754E+05	.1394E+02	.13[illegible]+01
113	.038	37.13	51	17	18	28	27			.3838E+05	.6822E+00	.1000E+01	.7191E+00	.3838E+05	.1920E+03	.718E-01
										.1921E+05	.5042E+00	.1921E+05	.5616E+00	.1000E+01	.4148E+02	.137E+01
114	.044	41.71	51	27	28	38	37			.3838E+05	.6823E+00	.1000E+01	.7155E+00	.3838E+05	.2494E+03	.730E-01
										.1798E+05	.4506E+00	.1000E+01	.5079E+00	.1798E+05	.4524E+02	.163E+01
115	.053	46.30	51	37	38	48	47			.3824E+05	.6923E+00	.1000E+01	.7214E+00	.3824E+05	.3335E+03	.720E-01
										.1792E+05	.4469E+00	.1000E+01	.4992E+00	.1792E+05	.5978E+02	.165E+01
116	.065	50.90	51	47	48	58	57			.3736E+05	.7126E+00	.1000E+01	.7374E+00	.3736E+05	.4366E+03	.857E-01
										.1729E+05	.4703E+00	.1000E+01	.5160E+00	.1729E+05	.7618E+02	.171E+01
117	.079	55.49	51	57	58	68	67			.3705E+05	.7423E+00	.1000E+01	.7628E+00	.3705E+05	.5839E+03	.777E-01
										.1691E+05	.5213E+00	.1000E+01	.5594E+00	.1691E+05	.1004E+03	.169E+01
118	.092	28.87	51	67	68	78	77			.3721E+05	.9343E+00	.1000E+01	.9367E+00	.3721E+05	.4269E+03	-.280E-01
										.1626E+05	.8773E+00	.1000E+01	.8819E+00	.1626E+05	.7716E+02	.129E+01
119	.100	29.97	51	77	78	88	87			.4091E+05	.9334E+00	.4091E+05	.9358E+00	.1000E+01	.5817E+03	-.115E+00
										.1869E+05	.8847E+00	.1869E+05	.8888E+00	.1000E+01	.1158E+03	.985E+00

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
120	.020	2.25	21	1	2			-.2653E-05							.1508E-19	
								-.2653E-05							.1508E-19	.226E+11
121	.020	2.63	21	3	4			-.4091E-05							.4186E-19	
								-.3637E-05							.3307E-19	.165E+11
122	.020	3.00	21	5	6			-.1592E-05							.7238E-20	
								-.1990E-05							.1131E-19	.302E+11
123	.020	2.63	21	7	8			-.5000E-05							.6253E-19	
								-.6364E-05							.1013E-18	.943E+10
124	.020	2.25	21	9	10			-.7428E-05							.1182E-18	
								-.6897E-05							.1019E-18	.870E+10
125	.020	2.70	21	11	12			-.4203E-05							.4540E-19	
								-.3097F-05							.2465E-19	.194E+11
126	.020	3.17	21	13	14			-.3387E-05							.3465E-19	
								-.2446E-05							.1808E-19	.245E+11
127	.020	2.85	21	15	16			-.3555E-05							.3436E-19	
								-.2300E-05							.1438E-19	.261E+11
128	.020	2.52	21	17	18			-.5452E-05							.7128E-19	
								-.2370E-05							.1347E-19	.253E+11
129	.020	2.56	21	19	20			-.3267E-05							.2600E-19	
								-.3500E-05							.2984E-19	.171E+11
130	.020	3.06	21	21	22			-.1169E-05							.3986E-20	
								-.9740E-06							.2768E-20	.616E+11
131	.020	3.60	21	23	24			-.1327E-05							.6035E-20	
								-.1161E-05							.4620E-20	.517E+11
132	.020	3.23	21	25	26			-.2215E-05							.1511E-19	
								-.2030E-05							.1269E-19	.296E+11
133	.020	2.85	21	27	28			-.1677E-05							.7624E-20	
								-.2305E-05							.1441E-19	.260E+11
134	.020	2.87	21	29	30			-.1354E-05							.5001E-20	
								-.1874E-05							.9589E-20	.320E+11
135	.020	3.43	21	31	32			-.1131E-05							.4179E-20	
								-.1479E-05							.7147E-20	.406E+11
136	.020	4.02	21	33	34			-.6675E-06							.1707E-20	
								-.7416E-06							.2108E-20	.809E+11

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
137	.020	3.61	21	35	36			-.1321E-05							.6008E-20	
								-.9909E-06							.3380E-20	.606E+11
138	.020	3.18	21	37	38			-.1032E-05							.3227E-20	
								-.1220E-05							.4508E-20	.492E+11
139	.020	3.17	21	39	40			-.8462E-06							.2165E-20	
								-.1128E-05							.3848E-20	.532E+11
140	.020	3.80	21	41	42			-.8648E-06							.2704E-20	
								-.7075E-06							.1810E-20	.848E+11
141	.020	4.45	21	43	44			-.2682E-06							.3050E-21	
								-.3353E-06							.4765E-21	.179E+12
142	.020	3.99	21	45	46			-.5978E-06							.1359E-20	
								-.5978E-06							.1359E-20	.100E+12
143	.020	3.51	21	47	48			-.2549E-06							.2174E-21	
								-.2124E-06							.1509E-21	.282E+12
144	.020	3.48	21	49	50			-.4711E-06							.7364E-21	
								-.2570E-06							.2191E-21	.233E+12
145	.020	4.16	21	51	52			-.2508E-06							.2495E-21	
								-.1433E-06							.8148E-22	.419E+12
146	.020	4.88	21	53	54			-.1530E-06							.1087E-21	
								-.1530E-06							.1087E-21	.392E+12
147	.020	4.37	21	55	56			-.2729E-06							.3103E-21	
								-.2900E-06							.3503E-21	.207E+12
148	.020	3.84	21	57	58			-.2717E-06							.2703E-21	
								-.2523E-06							.2331E-21	.238E+12
149	.020	3.79	21	59	60			-.1377E-06							.6850E-22	
								-.1180E-06							.5033E-22	.508E+12
150	.020	4.53	21	61	62			-.1071E-06							.4944E-22	
								-.1153E-06							.5734E-22	.520E+12
151	.020	5.30	21	63	64			.7036E-08							.2500E-24	
								.2111E-07							.2250E-23	.284E+13
152	.020	4.75	21	65	66			-.4710E-07							.1004E-22	
								-.5495E-07							.1367E-22	.109E+13
153	.020	4.18	21	67	68			-.2680E-07							.2856E-23	
								-.1340E-07							.7141E-24	.448E+13

MEMB	THICK	AREA	TYPE	MA	MB	MC	MD	SIGMA-X	SIGMA-Y	SIGMA-XY	EFSTR-1	EFSTR-2	EFSTR-3	EFSTR-4	ENERGY	MS
154	.020	4.15	21	69	70			-.2249E-08							.1998E-25	
								-.2249E-07							.1998E-23	.267E+13
155	.020	4.89	21	71	72			-.1906E-08							.1693E-25	
								-.1144E-07							.6095E-24	.525E+13
156	.020	5.65	21	73	74			.4289E-07							.9903E-23	
								.3959E-07							.8438E-23	.152E+13
157	.020	5.00	21	75	76			0.							0.	
								-.5591E-08							.1490E-24	.107E+14
158	.020	4.34	21	77	78			.2150E-08							.1909E-25	
								0.							0.	R

THE TOTAL ENERGY FOR LOADING CONDITION 1 IS .1632E+06 (U) .1632E+06(W)

THE TOTAL ENERGY FOR LOADING CONDITION 2 IS .1340E+06 (U) .1340E+06(W)

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JOINT	-X	-Y	-Z	FORCE-X	FORCE-Y	FORCE-Z	DISPL-X	DISPL-Y	DISPL-Z
1	63.500	90.000	1.125	205.000	-7380.000	926.000	-1.53156595E-01	-3.37310884E-01	1.50853503E+01
				351.000	-12600.000	1530.000	-5.12626745E-02	-3.47255404E-01	1.44830153E+01
2	63.500	90.000	-1.125	-205.000	7380.000	926.000	1.53156595E-01	3.37310884E-01	1.50853503E+01
				-351.000	12600.000	1530.000	5.12626745E-02	3.47255404E-01	1.44830153E+01
3	70.833	90.000	1.313	0.000	0.000	29.000	-1.68003833E-01	-3.64600013E-01	1.60655917E+01
				0.000	0.000	29.500	-4.39865366E-02	-3.51717978E-01	1.47515352E+01
4	70.833	90.000	-1.313	0.000	0.000	29.000	1.68003833E-01	3.64600013E-01	1.60655917E+01
				0.000	0.000	29.500	4.39865366E-02	3.51717978E-01	1.47515352E+01
5	78.167	90.000	1.500	-2800.000	-6960.000	1130.000	-2.11916646E-01	-4.68729580E-01	1.70648982E+01
				-2420.000	-6020.000	979.000	-6.02508514E-02	-4.36737902E-01	1.50078963E+01
6	78.167	90.000	-1.500	2800.000	6960.000	1130.000	2.11916646E-01	4.68729580E-01	1.70648982E+01
				2420.000	6020.000	979.000	6.02508514E-02	4.36737902E-01	1.50078963E+01
7	85.500	90.000	1.313	0.000	0.000	90.900	-1.87143175E-01	-3.97589127E-01	1.81325540E+01
				0.000	0.000	55.900	-4.46781197E-02	-3.61211376E-01	1.52850121E+01
8	85.500	90.000	-1.313	0.000	0.000	90.900	1.87143175E-01	3.97589127E-01	1.81325540E+01
				0.000	0.000	55.900	4.46781197E-02	3.61211376E-01	1.52850121E+01
9	92.833	90.000	1.125	-9870.000	-9780.000	1130.000	-1.84017545E-01	-3.99077236E-01	1.92778711E+01
				-4020.000	-3980.000	474.000	-4.66155072E-02	-3.38626999E-01	1.55597198E+01
10	92.833	90.000	-1.125	9870.000	9780.000	1130.000	1.84017545E-01	3.99077236E-01	1.92778711E+01
				4020.000	3980.000	474.000	4.66155072E-02	3.38626999E-01	1.55597198E+01
11	69.686	87.471	1.349	0.000	0.000	178.000	-1.68393505E-01	-3.77052511E-01	1.51335589E+01
				0.000	0.000	194.000	-4.12649435E-02	-3.66040446E-01	1.39597807E+01
12	69.686	87.471	-1.349	0.000	0.000	178.000	1.68393505E-01	3.77052511E-01	1.51335589E+01
				0.000	0.000	194.000	4.12649435E-02	3.66040446E-01	1.39597807E+01
13	76.097	84.851	1.586	0.000	0.000	214.000	-2.07968475E-01	-4.73436669E-01	1.51675480E+01
				0.000	0.000	175.000	-5.65356821E-02	-4.44512470E-01	1.34203495E+01
14	76.097	84.851	-1.586	0.000	0.000	214.000	2.07968475E-01	4.73436669E-01	1.51675480E+01
				0.000	0.000	175.000	5.65356821E-02	4.44512470E-01	1.34203495E+01
15	82.746	82.[illegible]33	1.427	0.000	0.000	253.000	-1.84877630E-01	-4.24220570E-01	1.52133131E+01
				0.000	0.000	157.000	-4.44832377E-02	-3.86436603E-01	1.28926685E+01
16	82.746	82.133	-1.427	0.000	0.000	253.000	1.84877630E-01	4.24220570E-01	1.52133131E+01
				0.000	0.000	157.000	4.44832377E-02	3.86436603E-01	1.28926685E+01
17	89.647	79.312	1.259	-5680.000	2320.000	1020.000	-1.66548864E-01	-3.88465419E-01	1.52287247E+01
				-1600.000	653.000	325.000	-3.91972815E-02	-3.49018684E-01	1.23217341E+01

JOINT	-X	-Y	-Z	FORCE-X	FORCE-Y	FORCE-Z	DISPL-X	DISPL-Y	DISPL-Z
18	89.647	79.312	-1.259	5680.000 1600.000	-2320.000 -653.000	1020.000 325.000	1.66548864E-01 3.91972815E-02	3.88455419E-01 3.49018684E-01	1.52287247E+01 1.23217341E+01
19	57.266	77.669	1.279	2310.000 5510.000	-946.000 -2250.000	723.000 1550.000	-1.50103626E-01 -5.17496373E-02	-3.22284307E-01 -3.21113405E-01	1.08415205E+01 1.07102222E+01
20	57.266	77.669	-1.279	-2310.000 -5510.000	946.000 2250.000	723.000 1550.000	1.50103626E-01 5.17496373E-02	3.22284307E-01 3.21113405E-01	1.08415205E+01 1.07102222E+01
21	63.992	74.920	1.532	0.000 0.000	0.000 0.000	314.000 347.000	-1.66550822E-01 -5.19120222E-02	-3.78524461E-01 -3.69407025E-01	1.09061029E+01 1.02800835E+01
22	63.992	74.920	-1.532	0.000 0.000	0.000 0.000	314.000 347.000	1.66550822E-01 5.19120222E-02	3.78524461E-01 3.69407025E-01	1.09061029E+01 1.02800835E+01
23	70.962	72.071	1.799	0.000 0.000	0.000 0.000	326.000 270.000	-1.92739326E-01 -5.85946987E-02	-4.65077469E-01 -4.41946078E-01	1.09137101E+01 9.79710603E+00
24	70.962	72.071	-1.799	0.000 0.000	0.000 0.000	326.000 270.000	1.92739326E-01 5.85946987E-02	4.65077469E-01 4.41946078E-01	1.09137101E+01 9.79710603E+00
25	78.191	69.116	1.617	0.000 0.000	0.000 0.000	338.000 213.000	-1.58157580E-01 -3.81557674E-02	-4.11940504E-01 -3.80672377E-01	1.09200094E+01 9.30515091E+00
26	78.191	69.116	-1.617	0.000 0.000	0.000 0.000	338.000 213.000	1.58157580E-01 3.81557674E-02	4.11940504E-01 3.80672377E-01	1.09200094E+01 9.30515091E+00
27	85.692	66.050	1.424	-4070.000 -1210.000	1660.000 496.000	902.000 311.000	-1.34810839E-01 -2.74908182E-02	-3.86352653E-01 -3.45888242E-01	1.08426840E+01 8.73800722E+00
28	85.692	66.050	-1.424	4070.000 1210.000	-1660.000 -496.000	902.000 311.000	1.34810839E-01 2.74908182E-02	3.86352653E-01 3.45888242E-01	1.08426840E+01 8.73800722E+00
29	51.032	65.339	1.433	1740.000 3990.000	-713.000 -1630.000	646.000 1310.000	-1.44133543E-01 -6.58601383E-02	-2.96809892E-01 -3.04333605E-01	7.28959532E+00 7.48498407E+00
30	51.032	65.339	-1.433	-1740.000 -3990.000	713.000 1630.000	646.000 1310.000	1.44133543E-01 6.58601383E-02	2.96809892E-01 3.04333605E-01	7.28959532E+00 7.48498407E+00
31	58.297	62.369	1.715	0.000 0.000	0.000 0.000	340.000 375.000	-1.51869900E-01 -5.51758216E-02	-3.44981522E-01 -3.37208706E-01	7.35327780E+00 7.12717600E+00
32	58.297	62.369	-1.715	0.000 0.000	0.000 0.000	340.000 375.000	1.51869900E-01 5.51758216E-02	3.44981522E-01 3.37208706E-01	7.35327780E+00 7.12717600E+00
33	65.826	59.291	2.012	0.000 0.000	0.000 0.000	352.000 291.000	-1.67953885E-01 -5.57287692E-02	-4.26140523E-01 -4.08652457E-01	7.33441024E+00 6.69261135E+00
34	65.826	59.291	-2.012	0.000 0.000	0.000 0.000	352.000 291.000	1.67953885E-01 5.57287692E-02	4.26140523E-01 4.08652457E-01	7.33441024E+00 6.69261135E+00

JOINT	-X	-Y	-Z	FORCE-X	FORCE-Y	FORCE-Z	DISPL-X	DISPL-Y	DISPL-Z
35	73.635	56.100	1.807	0.000	0.000	365.000	-1.31598297E-01	-3.76577355E-01	7.28909697E+00
				0.000	0.000	230.000	-3.10292813E-02	-3.49981104E-01	6.24447656E+00
36	73.635	56.100	-1.807	0.000	0.000	365.000	1.31598297E-01	3.76577355E-01	7.28909697E+00
				0.000	0.000	230.000	3.10292813E-02	3.49981104E-01	6.24447656E+00
37	81.738	52.787	1.590	-4250.000	1740.000	974.000	-1.09022988E-01	-3.57295406E-01	7.13862489E+00
				-1270.000	518.000	336.000	-1.82188447E-02	-3.19885301E-01	5.70271391E+00
38	81.738	52.787	-1.590	4250.000	-1740.000	974.000	1.09022988E-01	3.57295406E-01	7.13862489E+00
				1270.000	-518.000	336.000	1.82188447E-02	3.19885301E-01	5.70271391E+00
39	44.799	53.008	1.587	1820.000	-743.000	694.000	-1.27880637E-01	-2.49846167E-01	4.43438691E+00
				4160.000	-1700.000	1410.000	-6.63969885E-02	-2.62488420E-01	4.78522101E+00
40	44.799	53.008	-1.587	-1820.000	743.000	694.000	1.27880637E-01	2.49846167E-01	4.43438691E+00
				-4160.000	1700.000	1410.000	6.63969885E-02	2.62488420E-01	4.78522101E+00
41	52.603	49.818	1.898	0.000	0.000	365.000	-1.30665075E-01	-2.92457379E-01	4.49810288E+00
				0.000	0.000	402.000	-5.32217253E-02	-2.88831709E-01	4.51417272E+00
42	52.603	49.818	-1.898	0.000	0.000	365.000	1.30665075E-01	2.92457379E-01	4.49810288E+00
				0.000	0.000	402.000	5.32217253E-02	2.88831709E-01	4.51417272E+00
43	60.691	46.512	2.225	0.000	0.000	378.000	-1.39343038E-01	-3.65573242E-01	4.46771253E+00
				0.000	0.000	313.000	-5.06267100E-02	-3.53593407E-01	4.15762025E+00
44	60.691	46.512	-2.225	0.000	0.000	378.000	1.39343038E-01	3.65573242E-01	4.46771253E+00
				0.000	0.000	313.000	5.06267100E-02	3.53593407E-01	4.15762025E+00
45	69.079	43.083	1.997	0.000	0.000	392.000	-9.89174689E-02	-3.17475451E-01	4.39030667E+00
				0.000	0.000	247.000	-2.03986747E-02	-2.95573830E-01	3.77680570E+00
46	69.079	43.083	-1.997	0.000	0.000	392.000	9.89174689E-02	3.17475451E-01	4.39030667E+00
				0.000	0.000	247.000	2.03986747E-02	2.95573830E-01	3.77680570E+00
47	77.784	39.525	1.756	-4440.000	1820.000	1050.000	-7.54646233E-02	-3.05230452E-01	4.18278166E+00
				-1320.000	541.000	361.000	-4.84215299E-03	-2.71275070E-01	3.28711508E+00
48	77.784	39.525	-1.756	4440.000	-1820.000	1050.000	7.54646233E-02	3.05230452E-01	4.18278166E+00
				1320.000	-541.000	361.000	4.84215299E-03	2.71275070E-01	3.28711508E+00
49	38.565	40.678	1.742	1890.000	-773.000	742.000	-1.05739666E-01	-1.86543915E-01	2.32438568E+00
				4330.000	-1770.000	1500.000	-6.20971554E-02	-2.03050212E-01	2.71329020E+00
50	38.565	40.678	-1.742	-1890.000	773.000	742.000	1.05739666E-01	1.86543915E-01	2.32438568E+00
				-4330.000	1770.000	1500.000	6.20971554E-02	2.03050212E-01	2.71329020E+00
51	46.908	37.267	2.082	0.000	0.000	390.000	-1.03846760E-01	-2.20334079E-01	2.37590002E+00
				0.000	0.000	430.000	-4.74240236E-02	-2.21283761E-01	2.51245255E+00

JOINT	-X	-Y	-Z	FORCE-X	FORCE-Y	FORCE-Z	DISPL-X	DISPL-Y	DISPL-Z
52	46.908	37.267	-2.082	0.000	0.000	390.000	1.03846760E-01	2.20394079E-01	2.37590002E+00
				0.000	0.000	430.000	4.74240236E-02	2.21283761E-01	2.51245255E+00
53	55.555	33.732	2.438	0.000	0.000	404.000	-1.03947059E-01	-2.76969324E-01	2.33343889E+00
				0.000	0.000	334.000	-4.12753198E-02	-2.71403093E-01	2.23062811E+00
54	55.555	33.732	-2.438	0.000	0.000	404.000	1.03947059E-01	2.76969324E-01	2.33343889E+00
				0.000	0.000	334.000	4.12763198E-02	2.71403093E-01	2.23062811E+00
55	64.523	30.067	2.187	0.000	0.000	420.000	-6.26198328E-02	-2.33032555E-01	2.24239658E+00
				0.000	0.000	264.000	-8.20874278E-03	-2.16845636E-01	1.93907498E+00
56	64.523	30.067	-2.187	0.000	0.000	420.000	6.26198328E-02	2.33032555E-01	2.24239658E+00
				0.000	0.000	264.000	8.20874278E-03	2.16845636E-01	1.93907498E+00
57	73.830	26.262	1.922	-4640.000	1900.000	1120.000	-4.16609901E-02	-2.33004248E-01	2.00549282E+00
				-1380.000	565.000	386.000	7.39064873E-03	-2.04101821E-01	1.53137282E+00
58	73.830	26.262	-1.922	4640.000	-1900.000	1120.000	4.16609901E-02	2.33004248E-01	2.00549282E+00
				1380.000	-565.000	386.000	-7.39064873E-03	2.04101821E-01	1.53137282E+00
59	32.331	28.347	1.896	2290.000	-937.000	883.000	-7.97542288E-02	-1.15152948E-01	9.39622230E-01
				5300.000	-2170.000	1820.000	-5.29909849E-02	-1.34105539E-01	1.27310587E+00
60	32.331	28.347	-1.896	-2290.000	937.000	883.000	7.97542288E-02	1.15152948E-01	9.39622230E-01
				-5300.000	2170.000	1820.000	5.29909849E-02	1.34105539E-01	1.27310587E+00
61	41.214	24.716	2.265	0.000	0.000	413.000	-7.49426919E-02	-1.39588501E-01	9.71136133E-01
				0.000	0.000	458.000	-3.82899686E-02	-1.43064119E-01	1.12826391E+00
62	41.214	24.716	-2.265	0.000	0.000	413.000	7.49426919E-02	1.39588501E-01	9.71136133E-01
				0.000	0.000	458.000	3.82899686E-02	1.43064119E-01	1.12826391E+00
63	50.420	20.953	2.651	0.000	0.000	391.000	-7.05172863E-02	-1.74586532E-01	9.25682390E-01
				0.000	0.000	326.000	-3.24885445E-02	-1.75226816E-01	9.21641638E-01
64	50.420	20.953	-2.651	0.000	0.000	391.000	7.05172863E-02	1.74586532E-01	9.25682390E-01
				0.000	0.000	326.000	3.24885445E-02	1.75226816E-01	9.21641638E-01
65	59.967	17.050	2.376	0.000	0.000	368.000	-3.28161189E-02	-1.33328204E-01	8.32872079E-01
				0.000	0.000	233.000	-9.31470120E-04	-1.24469418E-01	7.28927885E-01
66	59.967	17.050	-2.376	0.000	0.000	368.000	3.28161189E-02	1.33328204E-01	8.32872079E-01
				0.000	0.000	233.000	9.31470120E-04	1.24469418E-01	7.28927885E-01
67	69.876	13.000	2.088	-3030.000	1240.000	804.000	-1.42984873E-02	-1.42639484E-01	5.96688240E-01
				-922.000	377.000	287.000	1.38097400E-02	-1.21961995E-01	4.29246873E-01
68	69.876	13.000	-2.088	3030.000	-1240.000	804.000	1.42984873E-02	1.42639484E-01	5.96688240E-01
				922.000	-377.000	287.000	-1.38097400E-02	1.21961995E-01	4.29246873E-01

JOINT	-X	-Y	-Z	FORCE-X	FORCE-Y	FORCE-Z	DISPL-X	DISPL-Y	DISPL-Z
69	25.166	14.173	2.073	3070.000	-520.000	1040.000	-4.22288799E-02	-4.09255801E-02	1.50235366E-01
				7160.000	-1210.000	2180.000	-3.46186534E-02	-6.02115756E-02	3.26853118E-01
70	25.166	14.173	-2.073	-3070.000	520.000	1040.000	4.22288799E-02	4.09255801E-02	1.50235366E-01
				-7160.000	1210.000	2180.000	3.46186534E-02	6.02115756E-02	3.26853118E-01
71	35.583	12.304	2.446	0.000	0.000	433.000	-4.53298717E-02	-5.96720986E-02	2.19892172E-01
				0.000	0.000	484.000	-2.79621842E-02	-6.11086852E-02	3.17463874E-01
72	35.583	12.304	-2.446	0.000	0.000	433.000	4.53298717E-02	5.96720986E-02	2.19892172E-01
				0.000	0.000	484.000	2.79621842E-02	6.11086852E-02	3.17463874E-01
73	46.181	10.403	2.827	0.000	0.000	370.000	-4.38219757E-02	-8.83518278E-02	2.58556125E-01
				0.000	0.000	310.000	-2.57691521E-02	-9.13801375E-02	2.71681902E-01
74	46.181	10.403	-2.827	0.000	0.000	370.000	4.38219757E-02	8.83518278E-02	2.58556125E-01
				0.000	0.000	310.000	2.57691521E-02	9.13801375E-02	2.71681902E-01
75	56.964	8.469	2.502	0.000	0.000	304.000	-1.98183986E-02	-5.92354093E-02	2.81201334E-01
				0.000	0.000	194.000	-2.48140918E-03	-5.57468272E-02	2.49187955E-01
76	56.964	8.469	-2.502	0.000	0.000	304.000	1.98183986E-02	5.92354093E-02	2.81201334E-01
				0.000	0.000	194.000	2.48140918E-03	5.57468272E-02	2.49187955E-01
77	67.938	6.500	2.169	-1370.000	262.000	446.000	-4.24715599E-03	-8.06102137E-02	1.86891812E-01
				-451.000	86.000	175.000	1.17652403E-02	-6.82306147E-02	1.24812534E-01
78	67.938	6.500	-2.169	1370.000	-262.000	446.000	4.24715599E-03	8.06102137E-02	1.86891812E-01
				451.000	-86.000	175.000	-1.17652403E-02	6.82306147E-02	1.24812534E-01
79	18.000	0.000	2.250	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.
80	18.000	0.000	-2.250	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.
81	30.000	0.000	2.625	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.
82	30.000	0.000	-2.625	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.
83	42.000	0.000	3.000	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.
84	42.000	0.000	-3.000	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.
85	54.000	0.000	2.625	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.

JOINT	-X	-Y	-Z	FORCE-X	FORCE-Y	FORCE-Z	DISPL-X	DISPL-Y	DISPL-Z
86	54.000	0.000	-2.625	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.
87	66.000	0.000	2.250	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.
88	66.000	0.000	-2.250	0.000	0.000	0.000	0.	0.	0.
				0.000	0.000	0.000	0.	0.	0.

Z-DISPLACEMENTS

NODE NO	LOAD CASE #1 ANALYZE	LOAD CASE #1 NASTRAN	LOAD CASE #2 ANALYZE	LOAD CASE #2 NASTRAN
1*	15.085	15.170	14.483	14.595
3	16.066	16.151	14.752	14.849
5	17.065	17.144	15.008	15.086
7	18.133	18.198	15.285	15.344
9	19.278	19.328	15.560	15.599
11	15.134	15.218	13.960	14.059
13	15.168	15.246	13.420	13.500
15	15.213	15.286	12.893	12.956
17	15.229	15.297	12.322	12.368
19	10.842	10.913	10.710	10.810
21	10.906	10.977	10.280	10.368
23	10.914	10.979	9.797	9.866
25	10.920	10.982	9.305	9.359
27	10.843	10.902	8.738	8.780
29	7.290	7.354	7.485	7.578
31	7.353	7.414	7.127	7.205
33	7.334	7.386	6.693	6.749
35	7.289	7.338	6.244	6.289
37	7.139	7.188	5.703	5.738
39	4.434	4.492	4.785	4.870
41	4.498	4.547	4.514	4.580
43	4.468	4.504	4.158	4.199
45	4.390	4.425	3.777	3.810
47	4.183	4.222	3.287	3.315
49	2.324	2.378	2.713	2.792
51	2.376	2.414	2.512	2.566
53	2.333	2.353	2.231	2.256
55	2.242	2.263	1.939	1.960
57	2.005	2.034	1.531	1.551
59	.940	.987	1.273	1.343
61	.971	.998	1.128	1.169
63	.926	.930	.922	.932
65	.833	.839	.729	.737
67	.597	.611	.429	.439
69	.150	.175	.327	.368
71	.220	.236	.317	.342
73	.259	.259	.272	.276
75	.281	.283	.249	.252
77	.187	.195	.125	.130

*Note: Results are given for nodes on the upper surface only.
The displacement pattern is identical on the lower surface.

TABLE 2: Results From ANALYZE and NASTRAN

*U.S.Government Printing Office: 1979 — 657-002/441